attunes it to the oscillations in deeper regions. Similarly my conclusion that $p_1 - \theta T_1$ is approximately zero, does not mean that the oscillations in region C are adiabatic. It is unlikely that the value θ_C near the photosphere is the same as the value θ_B at a much deeper level. My result is $p_1 - \theta_B T_1 = 0$, approximately, whereas the adiabatic condition is $p_1 - \theta_C T_1 = 0$.

I have tried to expose the fallacies unsparingly. May I therefore add that, as one who has lain fog-bound in these regions for eight years, I can well understand how they passed unnoticed.

On the State of Motion in the Galactic System.

By Bertil Lindblad, Ph.D.

(*Communicated by the Secretaries.*)

1. The aim of the present paper is to strengthen some arguments relating to a theory of the general state of motion of our stellar system which has been developed in a previous series of papers.* We assume that the stellar system has a general motion of rotation around an axis perpendicular to the galactic plane. The phenomenon of the "asymmetrical drift" of stellar velocities of great size, studied by Boss, Adams and Joy, Strömberg, Oort, and others, interpreted as due to a general decrease of the speed of rotation with increasing velocity dispersion, fixes the axis of rotation in the direction of the galactic longitude 330°. The direction of the rotation is retrograde, being from left to right for an observer situated to the north of the galactic plane. The direction towards the axis of rotation points very nearly towards the centre of distribution of the system of globular clusters according to Shapley's investigations.

The existence of such a general motion of rotation has received very strong support in a recent investigation by Oort † on the rotation effects in radial velocities and proper motions of distant galactic objects.

2. The interpretation of the phenomenon of the "asymmetrical drift" just mentioned implies that the stellar system may be divided formally into an infinite number of "sub-systems" of varying mean speed of rotation. The minimum of internal velocity distribution corresponds to the greatest speed of rotation, and thus defines the most rapidly rotating sub-system.

The formation of this particular state of equilibrium may be understood, at least partly, in the following way. Let us assume that the main part of the primitive stellar system once formed a mass of stellar and nebulous matter in a rather heterogeneous state of rotation, but on the whole very much flattened towards the galactic plane. As a possible origin of such a flattened system of high angular momentum we may

* *Arkiv för Matematik, Astronomi och Fysik*, 19 A, Nos. 21, 27, 35; 19 B, No. 7, 1925 and 1926. *Nova Acta Reg. Soc. Scient. Upsaliensis*, vol. extraord., ed. 1927.

† *B.A.N.*, **3**, 120, 1927.

picture a close encounter between two systems leading to a definite fusion. If W_1, T_1, W_2, T_2 are the potential and internal kinetic energy for the two systems, $\bar{T}_1$ and $\bar{T}_2$ the kinetic energy of translation relative to the centre of mass, for infinite distance between the two systems, and, finally, W_0 and T_0 the potential and kinetic energy of the combined mass, when it has reached a steady state after the encounter, we have,* according to Jeans,

$$\begin{aligned} W_0 &= W_1 + W_2 + 2(\bar{T}_1 + \bar{T}_2), \\ T_0 &= T_1 + T_2 - (\bar{T}_1 + \bar{T}_2). \end{aligned}$$

If we ignore the last terms of the right-hand members, it is clear that the respective potential and kinetic energies simply add together.

This process of combination is, of course, an indirect one. The decrease of potential energy due to the mutual approach of the two systems is compensated by an increase of internal potential energy at the cost of internal kinetic energy, and the loss in the latter sort of energy is supplied by the increase in kinetic energy of the two component systems following the original decrease of potential energy. The contribution to the internal kinetic energy from this source is likely to involve a high angular momentum in the plane of the encounter. Of course the fusion may also be partial in the way that part of the matter recedes again to infinity. Or the steady state may be "ordinarily" unstable, which probably means that the system will intermittently develop some sort of spiral structure † in which matter recedes to a finite distance and then returns. Such a cataclysmic motion will in this case continue, until the character of the steady state has been modified into a stable form.

Part of the primitive system may be assumed to concentrate into a central nucleus of nearly spherical form, while the rest arranges itself in a flattened figure of rotational symmetry representing a steady state under the "regular" or "hydrodynamic" forces. A great amount of matter in this part of the system is likely to move in circular orbits in the equatorial plane, and constitutes the most rapidly rotating sub-system, while other parts of stellar matter move in orbits of various inclinations towards this plane. A division into sub-systems of varying speed of rotation may therefore certainly be made in the early stages of the new system. It is likely that the physical development, the degeneration and regeneration, of individual stars may proceed under somewhat different conditions in different sub-systems, and that such circumstances are responsible for a great part of the physical differences which have in several cases been found to follow the division into "sub-systems."

There is, however, a certain process which must lead, though extremely slowly, to a successive ejection of matter from the very flattened sub-systems. We assume these sub-systems to be originally very rich in matter and to contain stars of various masses. Let us consider a certain point of the system and introduce a co-ordinate

* *Monthly Notices, R.A.S.*, **82**, 138, 1922.

† Cf. *Arkiv för Matematik* . . ., 19 A, No. 35, 20 A, No. 10; *Monthly Notices, R.A.S.*, March 1927.

system following the circular motion of rotation of a particle at this point with one of the axes steadily directed towards the centre of the system. The orbits in this co-ordinate system for relative velocities, which are small compared with the rotational velocity, are very strongly curved, and the dimensions of the orbits decrease with decreasing relative velocity (*cf.* § 5). We will assume that the dispersion of relative velocities at the point considered is small compared with the velocity of rotation. It follows, then, that a passage of two stars in approximately hyperbolic orbits can only be considered to take place for small distances between stars. The amplitude of variation in the force from a distant star during a revolution of the system will be small even in relation to the order of magnitude of the force. The forces from stars at great distances may therefore be considered to act on a star only by their combined action in a resultant of " regular " force. Though the region of effective " passages " around a star must be restricted in this way, we must assume that the distribution of the relative velocities will at each point tend to approach a Maxwellian distribution. The stars with smaller masses must then in the mean acquire higher velocities relative to the rotating frame, and the effective thickness of their distribution at right angles to the galactic plane must increase.

It may be shown at once that the stars acquiring higher relative velocities in this way will have a tendency to lag behind the stars in the galactic stratum in speed of rotation. We assume that a star belonging originally to the most rapidly rotating sub-system in the galactic plane is ejected from this plane by a small component of velocity normal to this plane, and we consider the projection of the orbit on the galactic plane. With increasing height the component of the attracting force towards the axis of rotation will decrease, the projection of the orbit must therefore widen, and the star accordingly recedes outwards. The area constant of the star's motion must under all circumstances fall below the area velocity of the circular orbits in the galactic plane for these greater distances from the axis. The mean angular velocity of rotation of the star will therefore be smaller than the angular velocity of the circular orbits for the same mean distance from the centre.

It is clear that if we assume an ejection of this kind to take place at all points of the " most rapidly rotating sub-system," members of sub-systems of lower speed of rotation will occur up to a somewhat larger radius in the galactic plane than the limiting radius for the first-mentioned sub-system. We need not assume, however, that the increase in effective radius with decreasing speed of rotation is of an order of magnitude comparable with the radius itself. This circumstance will be of importance in the following discussion.

3. In order to get a quantitative estimation of the relation between mean speed of rotation and internal velocity dispersion, we must consider more closely the conditions of the steady state under the action of the " regular " forces. We assume in accordance with a theorem of Jeans,* which has also been given independently by Charlier,† that

* *Monthly Notices, R.A.S.*, **76**, 70, 1915.
† *Lund Meddelanden*, I., No. 82, 1917.

the frequency function of the six-dimensional element of volume in the generalised space of position and velocity is of the form F (I_1, I_2), where I_1 and I_2 are the energy integral and the integral of areas on the galactic plane—*i.e.* in Jeans' system of notations,

$$I_1 = \Pi^2 + \Theta^2 + Z^2 - 2V, \quad I_2 = \varpi\Theta \quad . \quad . \quad (1)$$

ϖ, θ, and z are cylindrical co-ordinates, ϖ denoting the distance from the axis of rotation, θ an angular co-ordinate in the galactic plane, and z the vertical distance from this plane. Π, Θ, Z are the corresponding linear velocity components.

We may let the element $\varpi d\varpi d\theta dz d\Pi d\Theta dZ$ be formally represented by an element $dI_1 dI_2$ in a diagram that has I_2 and I_1 as axes. We consider the regions of finite $F(I_1, I_2)$ in this diagram. It is clear that we must have $I_1 < 0$, as the velocity of a star must fall below the velocity of escape. It is further clear that we have

$$I_1 > \frac{1}{\varpi^2} I_2^2 - 2V,$$

which means that for a given ϖ and z the region of finite frequency must lie between the axes $I_1 = 0$ and the parabola

$$I_2^2 = \varpi^2(I_1 + 2V) \quad . \quad . \quad . \quad . \quad (2)$$

The vertex of this parabola lies on the I_1-axis at a distance $I_1 = -2V$ from the origin. The parameter of the parabola for a given V is a maximum for points in the galactic plane.

It is of special interest to consider the envelope of the parabolas in the galactic plane, as this curve defines the entire area of finite $F(I_1, I_2)$. It may easily be verified that the envelope corresponds to the relation

$$I_2^2 = -\varpi^3 \frac{\partial V}{\partial \varpi},$$

and we then get from the signification of the parabolas according to the relations (1) and (2)

$$\Pi = 0, \quad Z = 0, \quad \Theta^2 = -\varpi \frac{\partial V}{\partial \varpi} \quad . \quad . \quad (3)$$

which corresponds to circular orbits in the galactic plane around the centre of the system.

We consider in fig. 1 the shape of the envelope for two different types of force. The full-drawn lines represent the case when the field of force is the same as if the mass could be dissected into homogeneous very flattened spheroids of the same equatorial radius which has been taken as unit of length. In this case $I_1 = 0$ corresponds to a circular motion along the equatorial boundary, and the envelope is a pair of straight lines $I_1 = \pm I_2\sqrt{3\pi GM} - 2V_0$, intersecting at the point $I_2 = 0$, $I_1 = -2V_0$, where $V_0 = \frac{3}{4}\pi GM$ is the function of force at the centre of the system. In the diagram we have put GM $= 1$.

The dotted curves represent the case when the same mass is concentrated in a central nucleus of spherical form, and are given by the equation $I_1 I_2^2 = -(GM)^2$. With decreasing radius of this nucleus the point of intersection with the I_1-axis is shifted towards $I_1 = -\infty$.

Oort's results, referred to above, indicate that the stellar system has a very considerable central condensation, so that we may assume the envelope curve to be of a character intermediate between the two curves of fig. 1.

Let us now consider the general distribution of relative stellar velocities as observed in the neighbourhood of the sun. We ignore for the moment the Kapteyn star-streaming, which will be considered later, and assume that for small velocities relative to the centroid, the velocity

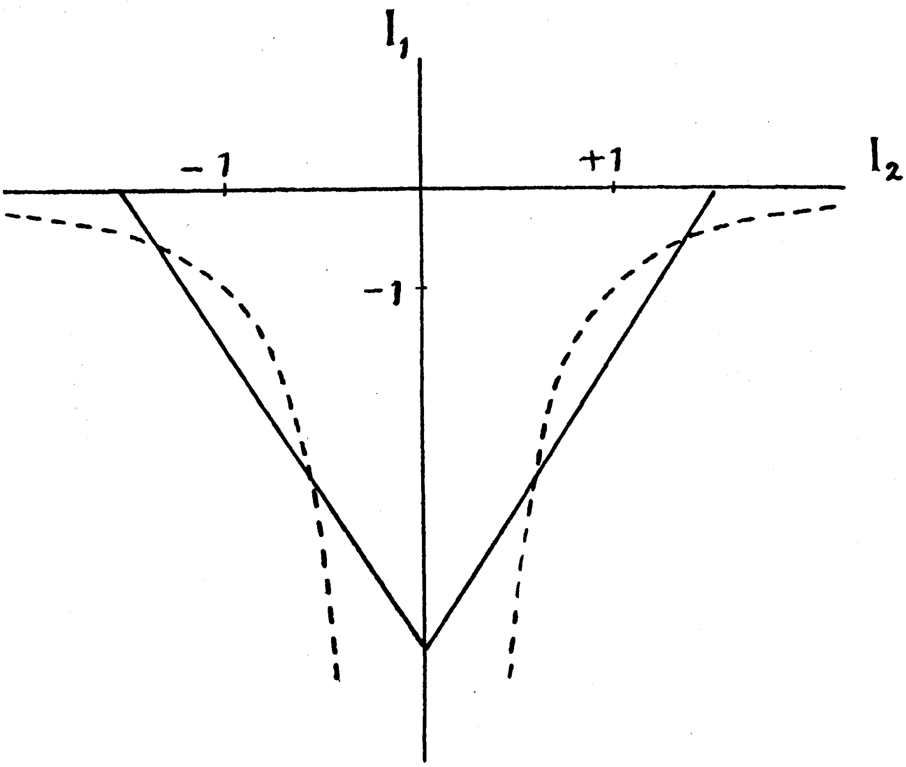

FIG. 1.

surfaces of equal frequency are nearly spherical. The velocity distribution as a whole for unit volume in our neighbourhood may then be pictured in the following way, which is shown schematically in fig. 2. We assume as an approximation to the observed distribution that in the velocity space the stars may be arranged in an infinite number of similar spherical distributions, which may be represented schematically by a series of velocity shells of decreasing radius with a uniform distribution on each shell. The radius of a shell is equal to the dispersion of the distribution. The area density on each shell increases at first rapidly with decreasing radius. The innermost shell is assumed to correspond to a circular motion around the centre.

In practice, however, the relation between velocity dispersion and mean motion has been found by determining the velocity dispersion and mean motion for groups of stars, defined either by the physical constitution of the members or in a way more directly associated with the velocities. Strömberg, for instance, makes a division according to apparent magnitude and proper motion ($m + 5 \log \mu$), and then uses the radial velocities for determining the mean drift and the velocity dispersion. We assume here that the relation between these characteristics of a group is the same as if the group constituted exactly a spherical shell in our sequence.

As a consequence of an advance towards equipartition of energy in the velocities relative to the centroid, we must expect the inner velocity shells to contain on the average stars of higher mass and luminosity than the outer ones. This seems to be partly confirmed by

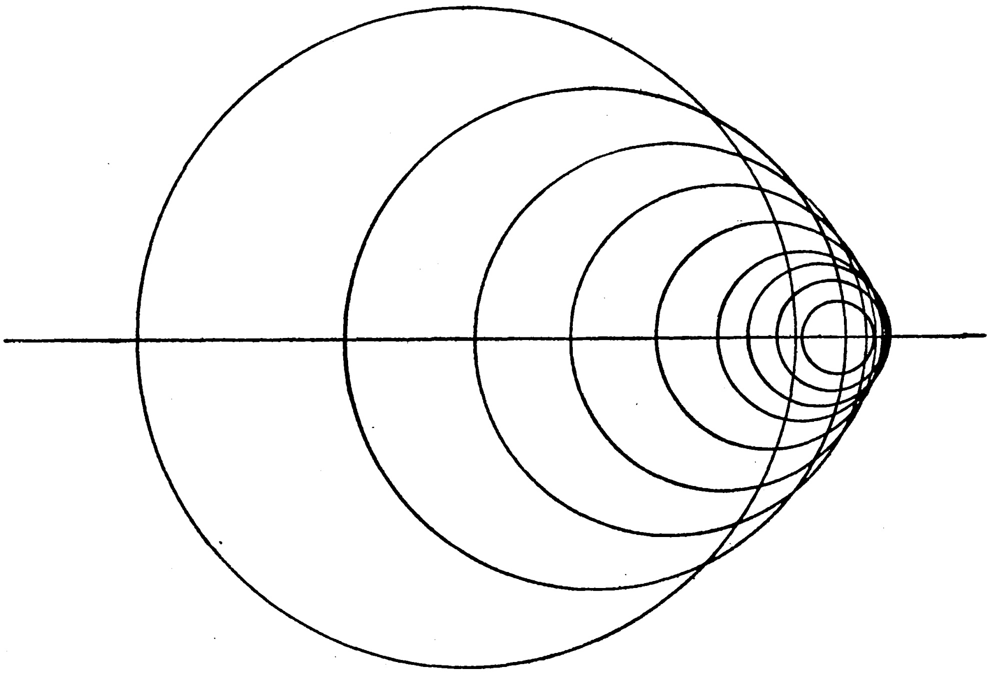

FIG. 2.—Schematic Dissection of the Observed Velocity Distribution into " Velocity Shells."

the observed data concerning the equipartition of energy in the translational motions relative to the centroid, but in some cases the equipartition process in question clearly cannot be of any significance—for instance, in relation to the high velocities within the sub-system constituted by the globular clusters. We must have to do in that case with a primeval cosmogonic cause the exact nature of which is, however, still completely obscure. On account of the general rotation of the system in the way discussed in § 2, the shells of greater radius have become shifted towards a lower mean speed of rotation. The character of the steady state under the " regular " forces does not, however, depend explicitly on the masses of the individuals.

A velocity shell has an immediate characteristic signification in the I_1, I_2-diagram of the system (fig. 3). It is represented simply by a uniform distribution along the part of a straight line

$$I_1 - kI_2 = c \quad . \quad . \quad . \quad . \quad (4)$$

which stretches across the characteristic parabola (2) valid for the point of the system considered. This may be verified immediately as follows. We have

$$\begin{aligned} I_1 - kI_2 &= \Pi^2 + \Theta^2 + Z^2 - 2V - k\varpi\Theta \\ &= \Pi^2 + (\Theta - \tfrac{1}{2}k\varpi)^2 + Z^2 - 2V - \tfrac{1}{4}k^2\varpi^2 = c. \end{aligned}$$

This corresponds to a spherical velocity shell with the mean velocity of rotation Θ_0 and with the radius R, where

$$\Theta_0 = \tfrac{1}{2}k\varpi, \quad R^2 = 2(V + \tfrac{1}{8}k^2\varpi^2) + c \quad . \quad . \quad (5)$$

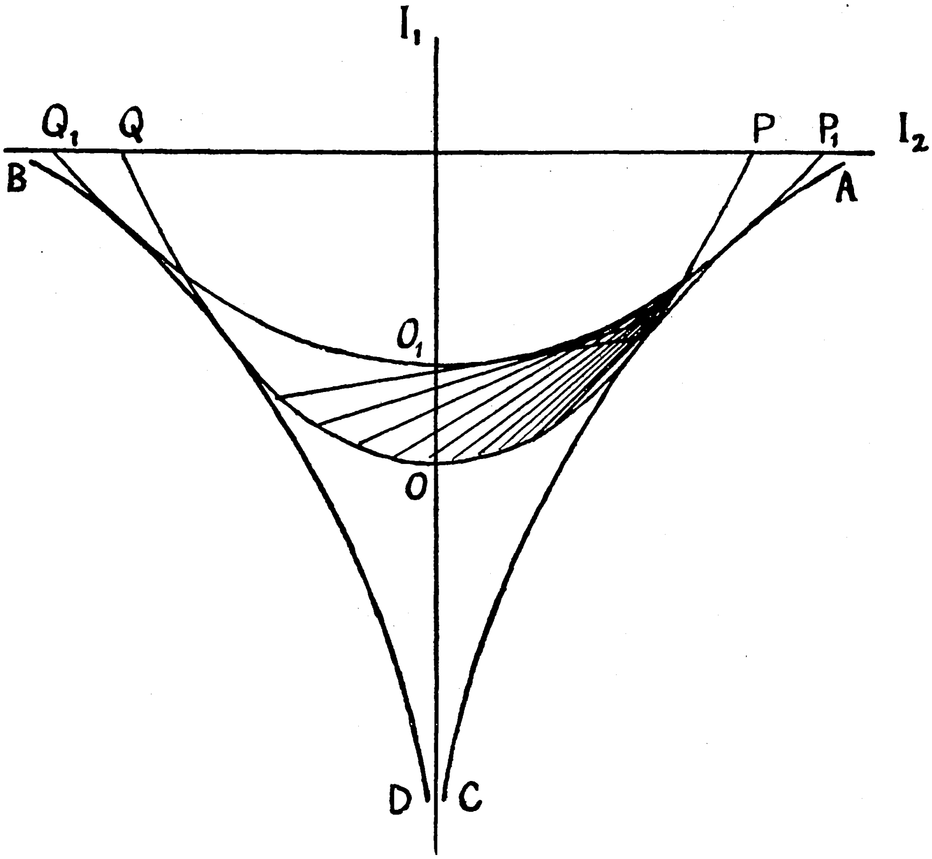

FIG. 3.—The Chords of the Parabola POQ correspond to the Velocity Shells of fig. 2.

which determines k and c in terms of Θ_0 and R. We remark that a point of the uniform distribution along one of the distribution lines in the I_1, I_2-diagram corresponds to all points in a narrow zone of a velocity shell, and thus does not correspond to an individual star.

The distribution of stellar velocities in our neighbourhood of space has thus been considered in the picture of a series of velocity shells which correspond to a series of chords of various inclinations in the characteristic parabola. As our point in the system lies nearly in the galactic plane, the characteristic parabola for this point will nearly touch the envelope curve. In fig. 3, AC and BD are the envelope curves and POQ is the characteristic parabola for our point of the system.

From the discussion in § 2 we derive the fundamental assumption that the different sub-systems, represented here by our series of velocity shells, have nearly the same maximum extension in the galactic plane. The curve $P_1O_1Q_1$ is the parabola corresponding to the limiting radius in this plane. The chords meeting $P_1O_1Q_1$ inside the parabola POQ must be tangents to $P_1O_1Q_1$, because otherwise the postulated uniform distribution on the chords within POQ would be interrupted. If our point of the system does not lie far from the limit, it is clear from the figure that we may assume as a general property of the chords that they are tangents to the limiting parabola $P_1O_1Q_1$. We now examine the consequences of this assumption for the relation between velocity dispersion and mean speed of rotation.

We introduce for convenience the notations

$$I_2 = x, \quad I_1 = y \quad . \quad . \quad . \quad . \quad (6)$$

and assume $\varpi = a$, $V = V_1$, at the limit of the system in the galactic plane. The equation for $P_1O_1Q_1$ then is

$$x^2 = a^2(y + 2V_1) \quad . \quad . \quad . \quad . \quad (7)$$

and for POQ

$$x^2 = \varpi^2(y + 2V) \quad . \quad . \quad . \quad . \quad (8)$$

The points of contact for the chords (4) on $P_1O_1Q_1$ may be denoted by x_1, y_1. The equation of one of these chords then is

$$xx_1 = \tfrac{1}{2}a^2(y + y_1 + 4V_1) \quad . \quad . \quad . \quad (9)$$

For the intersection between the straight line (9) and the parabola (8) we assume $x = x'$ and $x = x''$. From the significance of x in relation to the velocity shell,

$$x = \varpi\Theta,$$

and from the fact that the ends of the chord in POQ correspond to maximum and minimum of Θ in the velocity shell, we derive at once

$$\varpi\Theta_0 = \tfrac{1}{2}(x' + x''), \quad \varpi R = \tfrac{1}{2}(x' - x'') \quad . \quad . \quad (10)$$

Eliminating y and y_1 from equations (8), (9), and the relation

$$x_1^2 = a^2(y_1 + 2V_1),$$

we get for x' and x'' the expression

$$x = \frac{\varpi^2}{a^2}x_1 \pm \frac{\varpi}{a}\sqrt{2a^2(V - V_1) - x_1^2\left(1 - \frac{\varpi^2}{a^2}\right)} \quad . \quad (11)$$

If we insert in (10), and eliminate x_1, we find

$$\Theta_0^2 + \frac{\varpi^2}{a^2 - \varpi^2}R^2 = 2\frac{\varpi^2}{a^2 - \varpi^2}(V - V_1) \quad . \quad . \quad (12)$$

Introducing instead of R the velocity dispersion in a certain direction, *e.g.* the dispersion Z_0 at right angles to the galactic plane, we get

$$Z_0^2 = \tfrac{1}{3}R^2 \quad . \quad . \quad . \quad . \quad (13)$$

When ϖ is only slightly smaller than a, the right-hand member approaches to $-\varpi\frac{\partial V}{\partial \varpi}$, and thus we have

$$\Theta_0{}^2 + \frac{3\varpi^2}{a^2 - \varpi^2} Z_0{}^2 = -\varpi\frac{\partial V}{\partial \varpi} \quad . \quad . \quad . \quad (14)$$

We have $\Theta_0 = \text{max.}$ for $Z_0 = 0$. The "asymmetrical drift" is the quantity $(\Theta_0)_{\max} - \Theta_0$.

The expression (14) may be compared with the corresponding relation derived in previous papers. The expressions only differ by the coefficient 3 instead of 2 in the second term of the left-hand member. The present relation, however, has been derived *without special assumptions concerning the shape of the equipotential surfaces* and the dependence of star density on the potential $\Omega_k = V + \frac{1}{8}k\varpi^2$. The old expression rested on the assumption that for a certain sub-system we have near the equatorial boundary a constant density within a spheroidal surface, which cannot be exactly true, because the space density ought to increase with Ω_k. In the present case the characteristic relation between Θ_0 and Z_0 has been found to follow from the sole conditions that the galactic radius of maximum extension for different sub-systems is approximately constant, and that our point of the system is fairly near the limiting radius.

We assume $(\Theta_0)_{\max} = 300$ km./sec. as an approximate value from the drift velocity of the globular clusters, as computed by Lundmark * and by Strömberg.† The latter author's ‡ relation between asymmetrical drift and velocity dispersion is very well reproduced for

$$\frac{\varpi}{a} = 0{\cdot}84.$$

Our point of space should thus be situated 16 per cent. of the radius inside the galactic limit of the system. It would of course be premature to consider this result to be of more than a qualitative significance. It is evident, however, that the assumptions made have been reduced to a minimum, and that only observations of density and velocity distribution in fairly distant parts of the stellar system will make it possible to find starting-points for a refined theory.

It has to be remarked that we have not had to make any assumption about the frequency function $F(I_1, I_2)$ for the regions of the I_1, I_2-diagram below the characteristic parabola for our point of the system, *i.e.* for points of greater values of V, for instance, situated closer to the centre in the galactic plane. It is therefore not necessary to assume that the lines of uniform distribution continue as straight lines in those regions of the diagram.

When we proceed downwards along the right-hand envelope curve in fig. 3, we must expect to be forced to introduce new lines of distribution starting out in succession from the envelope curve in question.

* *Publ. Astron. Soc. of the Pacific*, **35**, 318, 1923.
† *Mount Wilson Contr.*, No. 292; *Astrophys. Journal*, **61**, 1925.
‡ *Ibid.*, No. 293.

In other words, we cannot expect all distribution lines to end as tangents of the parabola $P_1O_1Q_1$. The chords of a parabola POQ which correspond to the members of " the most rapidly rotating sub-system " for a point near the galactic plane must in the limit coincide with the common tangent of the envelope curve and the characteristic parabola, because they represent a circular motion in the galactic plane. According to the relation (3) this is identical with the condition that we shall always have for points in the galactic plane

$$(\Theta_0)^2{}_{\max} = -\varpi\frac{\partial V}{\partial \varpi} \quad . \quad . \quad . \quad . \qquad (15)$$

exactly as in the relation (14), which is thus valid generally for a vanishing value of Z_0.

It is of course not necessary to identify formally " sub-systems " and " distribution lines." The former conception may be considered as wider, embracing several distribution lines, or even being composed of successive parts of distribution lines, as in the case of the " most rapidly rotating sub-system," which contains the infinitely small chords of distribution of successive characteristic parabolas.

4. Oort's results indicate that the gravitational field in our neighbourhood of the system may be approximately governed by a combination of two central forces, one part K_1 due to a central spherical mass, and one part K_2 proportional to ϖ and thus possibly due to a mass of approximately spheroidal figure, or to the sum of a series of spheroidal distributions with the same centre and equatorial plane. The ratio $\frac{K_1}{K_2}$ was estimated to be 0·29. If we assume that K_2 is caused by a series of very flattened homogeneous spheriods of total mass M, and the force K_1 by a mass μM at the centre, we obtain for the ratio of the two masses

$$\mu = 4{\cdot}8.$$

The degree of concentration of the system is thus very considerable, and the " effective flattening " in what concerns the distribution of force in the galactic plane is fairly low. The system has therefore in all probability " ordinary " stability with respect to mass motions,* and the spiral orbits, which on occasional disturbance might be ejected from the system as " mother system," will differ very little from the circular boundary in the galactic plane. In this way the conditions do not seem to allow a development of clear spiral structure in our stellar system.

5. Among the stars of low relative velocities near the galactic plane we know from observed conditions, and may also account theoretically for, a tendency to group together in more or less definite local formations, star-clouds or local systems. The gravitational field must therefore be expected to show local irregularities. A close passage between two clouds, one exterior and one interior, on account of a slight change in angular speed of rotation following the difference in distance from the centre of the system, must tend to convert the difference in angular

* Cf. *Arkiv för Matematik* . . ., 20 A, No. 10, 1927.

speed of rotation into a more irregular two-dimensional motion in the galactic plane. Once such streamings originate, they will pursue their orbits for long times under the general field of force of the system. We may inquire about the form of these orbits relative to a rotating frame under a force of the kind just considered above.

Let us assume that the co-ordinate system ξ, η follows the maximum speed of rotation $(\Theta_0)_{\max}$ at the origin $\xi=0$, $\eta=0$, with the ξ-axis steadily directed towards the centre of the stellar system. For small motions in this co-ordinate system we obtain in the first approximation the elliptic orbits

$$\xi^2 + \left(\frac{\omega_1}{2\omega}\right)^2 \eta^2 = c^2 \quad . \quad . \quad . \quad . \quad (16)$$

where the origin has been chosen to coincide with the centre of the orbit. For the velocity components ξ', η' we get

$$\xi'^2 + \left(\frac{\omega_1}{2\omega}\right)^2 \eta'^2 = \omega_1{}^2 c^2 \quad . \quad . \quad . \quad (17)$$

In these formulæ ω is the angular speed of rotation of the most rapidly rotating sub-system at the point $\xi=0$, $\eta=0$. Putting

$$\beta = \frac{3\mu \mathrm{GM}}{\varpi^3},$$

we have further

$$\left.\begin{aligned} \omega^2 &= \beta\left[\frac{\pi}{4\mu}\left(\frac{\varpi}{a}\right)^3 + \frac{1}{3}\right] \\ \omega_1{}^2 &= 4\omega^2 - \beta \end{aligned}\right\} \quad . \quad . \quad . \quad (18)$$

When μ is small and the spheroidal mass M predominates, we have $\omega_1 = 2\omega$. When μ is great and the mass at the centre determines the character of the field of force, we get $\omega_1 = \omega$. In the former case the orbits are circular, in the latter case ellipses with axes in the ratio 2 : 1, with the longer axis at right angles to the radius vector of the system. Let us assume, in rough accordance with Oort's results, that ω corresponds to a velocity $\Theta_0 = 300$ km./sec. for $\varpi = 5000$ parsecs. We then get in the case of a maximum velocity ξ' in the direction of the radius vector of 30 km./sec. as value of the smaller axis of the ellipse

$$c = 390 \text{ parsecs.}$$

An idea of the dimensions of the relative orbits for ordinary relative velocities has thus been obtained.

We consider now a multitude of such motions traversing in all directions a certain small region of the system. Though in the co-ordinate system ξ, η the velocity in an orbit is a maximum in tangential direction, if μ has a considerable value, the *net stream velocity* V_T measured relative to the local velocity of rotation $(\Theta_0)_{\max}$ around the centre of the stellar system, and not as in the system ξ, η relative to a velocity of rotation Θ_0 corresponding to the value of ω for the centre of the particular relative orbit, is smaller in absolute amount than the maximum velocity $V_R = \xi'_{\max}$ in the direction of the radius.

We get

$$\frac{V_T}{V_R} = \frac{2\omega c - \varpi(\Delta\omega)_c}{c\sqrt{4\omega^2 - \beta}} \quad . \quad . \quad . \quad (19)$$

where $(\Delta\omega)_c$ is the change of $\omega = \frac{1}{\varpi}(\Theta_0)_{max}$ on an interval of the radius equal to the smaller axis c of a relative orbit.

The ratio in question decreases from 1·0 to 0·5, when μ increases from 0 to ∞. We get by differentiating the relation

$$\omega^2 = GM\left(\frac{3}{4}\frac{\pi}{a^3} + \frac{\mu}{\varpi^3}\right),$$

$$\varpi(\Delta\omega)_c = \tfrac{3}{2}\omega c \; \frac{\mu}{\frac{3}{4}\pi\left(\frac{\varpi}{a}\right)^3 + \mu}.$$

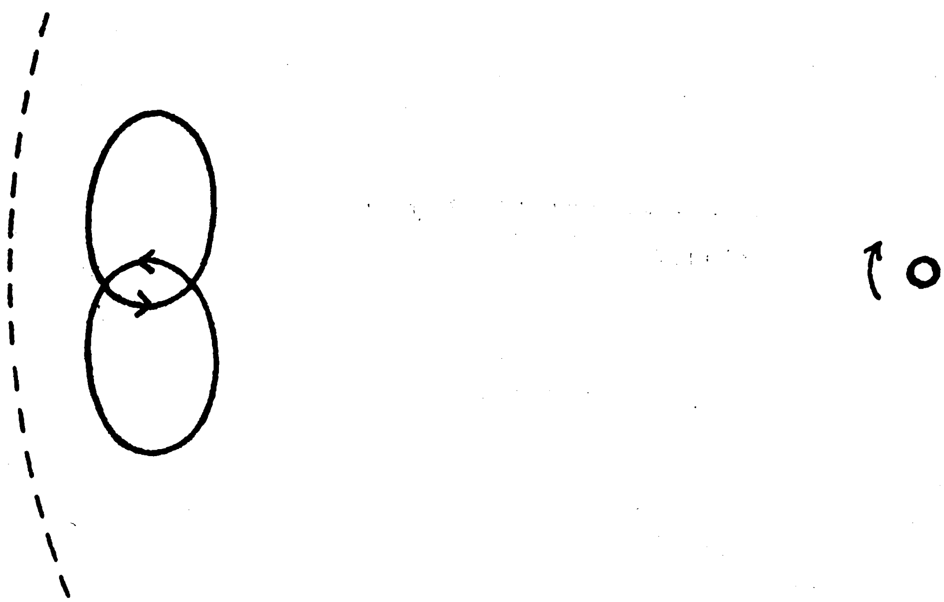

FIG. 4.—Relative Orbits in the Galactic Plane.

For $\mu = 4{\cdot}8$ and $\varpi = 0{\cdot}84a$ we then get

$$\frac{V_T}{V_R} = 0{\cdot}65.$$

The observed ratio of the axes of the velocity ellipsoid is thus approximately accounted for, if we assume that our point of space is traversed in all directions by streamings following similar orbits, carrying on the average an equal amount of matter per unit volume of the generalised space of position and velocity. Two orbits intersecting at opposite vertices have been pictured in fig. 4.

If we disregard the angle of about 20° which the mean observed vertex line forms with the direction towards the centre, the phenomenon of the Kapteyn star streaming seems to be in good accordance even quantitatively with the general state of motion of the galactic system as given by the present theory.