More trustworthy results can perhaps be obtained by calculating the path of the comet through Jupiter's sphere of activity, but without hypothesis this method will not accomplish our object; still such a work may possess some interest.

Pulkovo:
1910 *February.*

The Envelopes of Comet Morehouse (1908 *c*).

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CONTENTS.

§ 1. One of the most striking features of the photographs of Comet Morehouse, obtained with the 30-inch reflector at the Royal Observatory, Greenwich, is the clear definition of the parabolic envelopes, which often appear in and near the head of the comet. The very rapid changes of these envelopes form an important subject for study, and one that may be expected to throw light on the phenomena involved in the formation of the tail. When closely examined, the constitution of these envelopes is a much more perplexing problem than would at first sight appear; and the present paper does not claim to establish any definite conclusion. The great difficulty is their almost instantaneous formation, which seems to require the existence of a force of solar repulsion enormously greater than that generally accepted. I raised this question briefly in *Proc. Roy. Inst.*, 1909 March 26, and now give a fuller discussion.

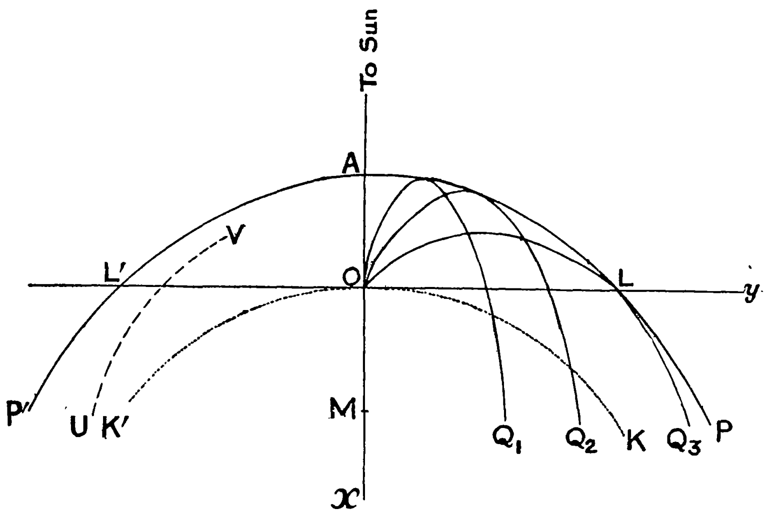
The present investigation forms part of a general discussion of the Greenwich photographs of the comet, which I have undertaken in conjunction with Mr. Davidson, and in this paper I am indebted to him for assistance in many ways. In view of the return of Halley's Comet, which may present opportunities of acquiring evidence on the doubtful points, it has seemed best not to delay the publication of this until the whole work was completed.

§ 2. *Measurement of the Photographs.*—The photographs were measured in a position-micrometer with a low-power eyepiece. The position of the radius vector from the Sun was computed, and the plate oriented so that it coincided with the direction of motion of the microscope by the micrometer-screw. The diaphragm in the microscope was ruled with two fine lines intersecting at right angles, each inclined at 45° to the direction of motion. This arrangement was originally adopted in order to take advantage of the property of the parabola,—"Any pair of tangents at right angles to one another meet on the directrix." The point of intersection of the lines could thus be placed successively on the focus (nucleus), vertex and directrix, giving two independent determinations of the focal length of the envelope. But in addition, this diaphragm proved to be a very convenient device for measuring the two rectangular coordinates with one micrometer-screw; if a point is brought successively under the two lines, half the sum of the readings gives its x coordinate, and half the difference gives its y coordinate.

In almost every case the nucleus showed as a well-defined stellar point, and the error of setting on the nucleus was practically nil.

§ 3. *Fountain-Theory of Envelopes.*—It is well known that when a great number of particles are projected from a point with equal velocities in all directions under gravity, the envelope of their paths is a paraboloid having the point of projection as focus. There seems to be little doubt that cometary envelopes are formed in an analogous manner. In this case the repulsion from the Sun takes the place of gravity, the particles are supposed to be projected from the nucleus with equal velocities in all directions in the hemisphere towards the Sun (those in the hemisphere away from the Sun have no part in forming the envelope), and the envelope is a paraboloid having the nucleus as focus and forming a kind of dome over it. In the diagram, $O\,Q_1$, $O\,Q_2$ and $O\,Q_3$ are the paths of individual particles, and all touch internally the parabola $P\,A\,P'$, which is the envelope. This theory, which we shall call the *fountain-theory*, was adopted by Bessel, Bredichin, Bond, and others. It will be noticed that according to this theory at successive instants different particles form the envelope; the change of position of an envelope is not equivalent to the corresponding material displacement. In a later section I shall consider the evidence for this theory of the envelopes; but meanwhile it may be stated that the difficulties in the way of any other explanation appear to be overwhelming.

The envelopes, which appear in the photographs, are for the most part sharply-defined fine curves, and differ rather markedly from the appearances to which the term is usually applied in other comets, although they are probably analogous. It is possible to use the word "envelope" in two rather different senses, either as a boundary beyond which little or no material is projected, or as a surface of exceptional density of the material. It is the latter feature that particularly characterises the envelopes of Comet Morehouse, whereas in the case of other comets, such as Donati, Coggia, 1910 *a*, in which the envelopes have been particularly noticed, the term has been used in the former sense to denote a boundary with little or no increase in density of the material. Both kinds of envelope are accounted for by the fountain-theory; but it would seem that in Comet Morehouse each fine envelope corresponds to a distinct and separate explosion, whilst in the



other comets the explosions were so numerous that only a general average effect was observed.

§ 4. *Transitory Character of the Envelopes.*—It is for this reason that we were able to observe in Comet Morehouse, what I believe had not been noticed in any previous case, the extremely transitory character of the individual envelopes. In Table I. are given measures of envelopes on October 27 and 30, these being dates when the most abundant data are available, but the same thing can be seen on any night when envelopes were shown. Where possible, two independent measures made respectively at the apex and latus rectum of the envelope are given. It will be seen that the envelope always begins to collapse immediately after its first formation, and shrinks continually. *I know of no case where an envelope expands.* After three or four hours it has completely degenerated, the part between the nucleus and the Sun being lost in the uprushing material for new envelopes, whilst that behind the nucleus mingles with the tail. It is also remarkable that the envelope becomes more strongly defined as it contracts.

TABLE I.

Focal Lengths of Envelopes, showing the rapidity with which the force of the explosion declines.

No. of Envelope.	Measures at	Time of Photograph, 1908 October 27.									
		$6^h 41^m$.	$7^h 5^m$.	$7^h 23^m$.	$7^h 44^m$.	$8^h 4^m$.	$8^h 23^m$.	$9^h 3^m$.	$9^h 32^m$.	$10^h 2^m$.	$10^h 28^m$.
		′	′	′	′	′	′	′	′	′	′
1	A	·80	·74	·66	·47	·44	·39	·39	·33	·30	...
	L	·76	·62	·57	·49	·42	·36	·33	·30	·25	...
2	A	1·46	1·30	·97	·86	·60	...	...	...	...	...
	L	1·34	1·09	1·02	·86	·61	·52	·46	...	...	...
4	A	...	...	...	...	·97	·74	·67	·64	...	...
	L	...	...	...	...	...	...	...	·68	·64	·54
5	A	2·3	2·2	1·56	...	1·28	1·06	·99	...	...	...
	L	...	...	...	...	...	1·04	·94	...	...	...
6	A	...	...	...	...	...	1·90	1·62	1·38	1·22	1·15
	L	...	...	...	...	...	...	1·48	1·38	1·12	...

No. of Envelope.	Measures at	1908 October 30.								
		$6^h 21^m$.	$6^h 37^m$.	$7^h 2^m$.	$7^h 21^m$.	$7^h 47^m$.	$8^h 4^m$.	$8^h 19^m$.	$8^h 52^m$.	$9^h 19^m$.
		′	′	′	′	′	′	′	′	′
1	A	1·26	1·16	·92	·63	...	·49	...	...	...
	L	1·16	1·07	·85	·74	·63	·57	·39	...	...
2	A	·60	·57	·44	·38	...	...	...	...	...
	L	·62	·55	·45	·40	·34	·26	·22	...	...
3	A	1·66	1·52	1·34	1·22	·99	·88	·81	·66	·50
	L	...	1·48	1·34	1·27	1·04	·89	·76	·65	·46
4	A	...	...	...	...	1·55	1·50	1·41	·90	...
	L	...	...	...	...	1·69	1·64	1·29	·90	·65

It seems clear that each envelope corresponds to a single outburst, strongest at first, *i.e.* projecting matter with greatest velocity, and gradually declining in force, though not necessarily in the volume of matter emitted, the whole outburst lasting three or four hours. Several outbursts are generally taking place simultaneously, presumably at different points of the nucleus. The question may be raised whether all the matter experiences the same repulsive force from the Sun. Where two or more envelopes exist simultaneously, it might be thought that they correspond to matter undergoing different amounts of repulsion, as in Bredichin's theory of the different tails. But I think that the size of the envelope depends chiefly on its age; on any day the envelopes form successively at more or less the same height, and as each contracts, another is formed outside; thus, when several envelopes exist simultaneously, they may be regarded as showing successive stages of similar outbursts. Nevertheless, the envelopes on different nights often vary very greatly in size and general character, and it is probable that there are large variations in the repulsive force.

§ 5. *Time of formation of an Envelope.*—When an outburst takes place, some time must elapse before the ejected material reaches the required position and forms the envelope. Further, the envelope cannot all be formed at the same instant; the part at the apex forms first, and the envelope gradually spreads, and extends further and further away from the Sun.

Let V be the velocity of ejection, g the accelerating force of repulsion from the Sun.

Take the origin at the nucleus, and the axis of x along the radius vector away from the Sun (see diagram, p. 444).

After a time t, the position of a particle ejected at an angle $(180° - \alpha)$ to Ox, is

$$x = -Vt\cos\alpha + \tfrac{1}{2}gt^2$$
$$y = Vt\sin\alpha,$$

whence, eliminating α,

$$\tfrac{1}{4}g^2t^4 - (V^2 + gx)t^2 + x^2 + y^2 = 0 \quad . \quad . \quad . \quad (1)$$

and the condition for real roots is

$$y^2 < \frac{2V^2}{g}\left(x + \frac{V^2}{2g}\right).$$

Thus the envelope is the parabola

$$y^2 = 4a(x + a), \quad \text{where} \quad a = \frac{V^2}{2g}.$$

(In the figure this parabola is P L A L′ P′ and a = OA.)

The time taken to reach any point on the envelope is derived from equation (1). Remembering that for points on the envelope the equation has equal roots, we obtain at once

$$\tfrac{1}{2}gt^2 = \sqrt{(x^2 + y^2)} = x + 2a \quad . \quad . \quad . \quad (2)$$

and

$$t = \sqrt{\frac{2}{g}} \cdot \sqrt[4]{(x^2 + y^2)} = \sqrt{\frac{2}{g}}\,\sqrt{x + 2a} \quad . \quad . \quad (3)$$

The time is thus equal to that taken by a particle to fall from rest on the directrix to the point considered, under the repulsive force.

For example, putting $x = -a$ for the apex A, and $x = 0$ for the latus rectum L. The time from the nucleus to the apex $= \sqrt{\frac{2a}{g}}$; and the time to the latus rectum is $\sqrt{\frac{2a}{g}} \times \sqrt{2}$. We cannot observe either of these directly; but we can, by noticing the growth of the envelope on successive photographs, estimate their difference, *i.e.* how long after the first appearance of the envelope at A it has spread as far as L and L′. This gives $\sqrt{\frac{2a}{g}}(\sqrt{2} - 1)$; and, as we can measure a, g can be found.

In general, the paraboloidal envelope has its axis inclined to the plane of the photograph, so that what we see on the photograph is not a central section of the paraboloid. It can be shown that the apparent outline will always be a parabola, having the nucleus as focus. The focal length *a*, however, is not foreshortened, but is greater in the parabola than in the paraboloid in the ratio sec I, where I is the inclination of the axis of the paraboloid to the plane of the photograph. For most purposes it is sufficient to ignore motions in the line of sight, considering only the particles projected in the plane of the photograph, and the component of the Sun's repulsion resolved in that plane. This leads to the same result. But it is useful to remember that the apex of the parabola does not correspond to the apex of the paraboloid; for instance, when I is 45°, the apex of the parabola is a point on the focal plane of the paraboloid. In the case of Comet Morehouse, during the period under consideration, I was never less than 43°, and sometimes as much as 60°. We naturally think of the apex of an envelope shown in the photograph as lying between the nucleus and the Sun, but, owing to the inclination, this was not usually the case. We see practically nothing in these photographs of the paraboloid between the nucleus and the Sun, that part having nothing to do with the parabolic outline which is the two-dimensional appearance of the envelope.

§ 6. It is found that the photographs of Comet Morehouse indicate generally a remarkably rapid formation of the envelope. Very soon after the first appearance of envelope formation at the vertex, the whole envelope, extending some distance beyond the latus rectum, appears to be complete. It is not very easy to put in a convincing form the evidence on which this assertion rests. The extremity of an envelope is seldom abruptly defined, and its apparent extent depends on the excellence of the photograph; further, the envelopes tend to become denser and more clearly defined as they contract, and from this cause may be traced farther. I give, however, a few instances in which the evidence can be definitely stated, and appears to me unmistakable; these are supported by a great deal of less precise confirmatory evidence.

1. *Oct.* 18.—An envelope on the fore side of the comet (*i.e.* in the direction in which it is moving). On plate 3173 (7^h 58^m) there is no sign of this envelope; a little hazy material is seen where the vertex would be, but there is no ridge or distinct boundary. On plate 3174, 57 minutes later, the envelope has formed; it is very distinct, fine and regular, one of the most perfect examples of an envelope that I have come across. It extends clearly beyond the nucleus to $x = 1'.1$, and is traceable faintly to about $x = 2'$. On plate 3175, 55 minutes later, the envelope is less sharply defined, but extends clearly to $x = 1'.9$, and can be traced farther. The values of *a* (focal length) are, on plate 3174, $0'.56$; on plate 3175, $0'.25$.

This evidence of extremely rapid formation is the more convincing as, of the three photographs (each having 10 minutes' exposure), 3173 is much the densest and shows the most detail generally, and 3174 is considerably better than 3175. Thus the absence of the envelope on 3173 cannot possibly be attributed to inferiority of the photograph.

2. *Oct.* 15.—Envelope on rear side.

Plate 3167. $7^h\ 3^m$, invisible.
3168. $7^h\ 24^m$, faint, but formed as far as $x = 1'{\cdot}2$.
3170. $8^h\ 20^m$, well defined, extends at least to $x = 5'{\cdot}6$.

Values of a are $0'{\cdot}72$ and $0'{\cdot}48$ on 3168 and 3170 respectively. The photographs are of practically equal density and definition.

3. *Oct.* 15.—Another envelope on rear side.

Plate 3170. $8^h\ 20^m$, envelope just formed at vertex, $a = 1'{\cdot}18$.
3171. $9^h\ 24^m$, envelope extends at least $2'{\cdot}5$ beyond its latus rectum, $a = 0'{\cdot}95$.

4. *Oct.* 27.—Envelope chiefly on fore side, but also extending some distance on rear side.

Plate 3203. $8^h\ 4^m$, haze near where apex of envelope would be, but no indication of a definite boundary.
3204. $8^h\ 23^m$, boundary distinct; the envelope seems to be just forming at the apex $a = 1'{\cdot}90$.
3206. $9^h\ 3^m$, envelope clearly seen as a ridge, and extends nearly to the latus rectum, $a = 1'{\cdot}55$.
3207. $9^h\ 32^m$, envelope fully developed and extends just beyond the latus rectum, $a = 1'{\cdot}38$.

5. *Oct.* 30.—Envelope chiefly on rear side; rather indistinct.

3220. $7^h\ 21^m$, invisible.
3221. $7^h\ 47^m$, envelope formed for a short arc at vertex, $a = 1'{\cdot}5$.
3222. $8^h\ 4^m$, envelope nearly reaches latus rectum, $a = 1'{\cdot}45$.
3223. $8^h\ 19^m$, envelope reaches beyond latus rectum, $a = 1'{\cdot}42$.

6. *Oct.* 1.—Envelope on rear side.

3127. $7^h\ 53^m$, quite invisible.
3128. $8^h\ 35^m$, extends $2'$ beyond the nucleus; for this envelope, a is very small, about $0'{\cdot}1$.

The exposures were 30^m and 15^m respectively, and it is just possible that the long exposure of 3127 may render the envelope too indistinct on that plate to be visible, but I think that is unlikely.

§ 7. *Calculation of the Repulsion.*—The first example given above is the most decisive. Let us examine to what lower limit of the value of g it leads. a is variable, but we may take $a = 0'.7$ as being probably about the mean value of a between plates 3173 and 3174. If we assume that the envelope began to form immediately after 3173 was exposed, and that on 3174 it extended to $x = 1'.1$ (ignoring the fainter continuation), the resulting value of g will certainly be a lower limit. If t be the time from the nucleus to the apex in hours, equation (2) gives

$$\tfrac{1}{2}gt^2 = 0'.7$$

$$\tfrac{1}{2}g\left(t + \frac{19}{20}\right)^2 = 1'.1 + 2 \times 0'.7 = 2'.5,$$

whence eliminating t, $g = 1'.23$ per hour per hour; converting into kilometres and allowing for the inclination of the radius vector to the plane of the plate, the full force of solar repulsion is not less than 81,500 kilometres per hour per hour. But the solar gravitation is only 34·6 km./hr.^{2} on this date. Thus the repulsion appears to be more than 2300 times the ordinary gravitational attraction. The other examples lead to the following lower limits for g, and μ (the ratio of the solar repulsion to the gravitational attraction).

Ex. 2.	(Oct. 15)	$g > 2'.0$	per hour	per hour;	$\mu >$	4,000
3.	(Oct. 15)	$g > 1'.8$	,,	,,	$\mu >$	3,500
4.	(Oct. 27)	$g > 0'.11$	,,	,,	$\mu >$	180
5.	(Oct. 30)	$g > 4'.1$	,,	,,	$\mu >$	6,500
6.	(Oct. 1)	$g > 6'.1$	,,	,,	$\mu >$	19,000

These values are of course much greater than the values ordinarily found. In Bredichin's researches μ was not greater than 36; subsequent discussions of observations by Jaegermann and others have led to values of μ up to about 80. Direct measurement of the motion of the tail particles of Comet Morehouse leads to values of μ ranging under normal conditions up to rather more than 100; and on October 1 and 2, when the motion was quite exceptionally rapid, μ may have been as great as 800. But the above results seem to show that the repulsive forces acting on the envelope-material are generally much greater still, and it will be necessary to carefully consider whether there is any possible escape from this conclusion.

§ 8. *Second Method.*—There is another method of testing the rapid formation of the envelopes, which leads to more precise numerical results, provided the simple theoretical conditions we have stated are rigorously fulfilled. If the eruption of material is not in a steady state, the form of the envelope will no longer be a paraboloid but will be modified. It has been shown that as we pass along the envelope from the apex, the matter forming the envelope at the instant has taken successively longer intervals to travel from the nucleus to the present position, and therefore was emitted at correspondingly earlier stages of the explosion. But as the force of the explosion was greatest at first and gradually declined, the earlier the time of emission the larger will be the paraboloid to which this portion of the envelope belongs. If we still write the equation of the envelope

$$y^2 = 4a(x+a)$$

$a = \frac{v^2}{2g}$ is variable, that value of v being taken which corresponds to the instant at which the matter now at (x, y) left the nucleus. The successive elements of the curve may thus be considered to form parts of parabolas, of which the focal length gradually increases with x. The shape will in fact be hyperbolical.

This is actually found to be the case on measuring the photographs. I have measured a great many envelopes to determine their precise shape, and find that along any envelope, except occasionally between the apex and latus rectum, a steadily increases with x. I select one or two examples to make this clear.

Table II.

Hyperbolic shape of envelopes, shown by change of the value of a at different parts of the curve.

(Unit = 2′·5.)

Plate 3139. Oct. 2. x	a	Plate 3164. Oct. 14. x	a	Plate 3201. Oct. 27. x	a	Plate 3229. Nov. 3. x	a
− ·12	·326	+ ·28	·083	+ ·08	·402	+ ·45	·204
+ ·41	·432	1·17	·132	·86	·409	·86	·218
+1·34	·597	1·81	·164	2·26	·441	1·52	·223
		4·05	·252				

Between the vertex and latus rectum the value of a sometimes decreases slightly, as Table I. shows, but this is not necessarily a real effect. The envelopes at the vertex are always close together, very dense and often confused. In these circumstances, seeing that we are measuring an outer limit, there may be a tendency to exaggerate the size near the vertex. The measures near and beyond the latus rectum seem to be much superior in accuracy, as the envelope is there clearly separated from those immediately inside or outside it. It has already been pointed out that the part of the *parabola* between the vertex and latus rectum does not correspond to the part of the *paraboloid* between the nucleus and the Sun, so that it is unlikely that the shape of this part of the envelope follows a different law from the rest.

Now if, taking the same envelope on two different photographs, we select two points, one on each of the photographs, for which a is the same, presumably the matter at these two points left the nucleus with the same velocity V, and therefore at the same instant. This is sufficient to enable us to determine g. Thus, on September 17, there is a very sharply defined envelope on the fore side of the comet, for which I obtained the following measures of arbitrary points on the curve.

1908 Sept. 17 (Unit = 2′·5).

Plate and Time.	x.	y.	$\sqrt[4]{x^2+y^2}$.	a.
3094	− ·043	·192	·44	·120
8^h 12^m	+ ·290	·543	·78	·163
	1·107	1·038	1·23	·206
3095	+ ·017	·159	·40	·072
9^h 6^m	·450	·528	·83	·122
	1·021	·831	1·15	·148
	2·593	1·585	1·74	·223
	3·616	1·965	2·03	·250
3096	+ ·179	·243	·55	·061
10^h 4^m	·868	·636	1·04	·104
	1·802	1·019	1·44	·134
	4·067	1·974	2·13	·227

Now equation (3) shows that for a given instant of emission, that is for a given value of a, $\sqrt{x^2+y^2}$ increases proportionately to the time. From the above table, I find the increase is ·40 per hour, *i.e.* ·35 in the interval between 3094 and 3095, and ·38 in the interval 3095 to 3096. That the data are entirely consistent with this may be seen from the following:—

$$\text{Putting } \tau = \sqrt[4]{x^2+y^2} + \cdot 40(9^h\ 6^m - t)$$

we have

a.	τ.	Plate.
·061	·17	3096
·072	·40	3095
·104	·66	3096
·120	·79	3094
·122	·83	3095
·134	1·06	3096
·148	1·15	3095
·163	1·13	3094
·206	1·58	3094
·223	1·74	3095
·227	1·75	3096
·250	2·03	3095

showing a regular change of a with τ, irrespective of the particular photograph.

Again from equation (3)

$$\cdot 40 = \sqrt{\frac{g}{2}},$$

whence $g = 0{\cdot}32$ per hour per hour
$= 0'{\cdot}80$,, ,,

and allowing for foreshortening the full solar repulsion

$= 90{,}000$ kilometres per hour per hour.

This corresponds to $\mu = 4000$.

The following are the results of similar determinations on other nights:—

Reference No.	Date.	g.	μ.
1	Oct. 2	$0'{\cdot}243$ per hour per hour	730
2	,,	$0'{\cdot}310$,, ,,	930
3	Oct. 14	$3'{\cdot}80$,, ,,	8,000
4	Oct. 15	$5'{\cdot}20$,, ,,	10,500
5	Oct. 27	$7'{\cdot}20$,, ,,	12,000
6	,,	$6'{\cdot}5$,, ,,	11,000
7	,,	$9'{\cdot}8$,, ,,	16,000
8	,,	$7'$,, ,,	12,000
9	Nov. 3	$0'{\cdot}45$,, ,,	700
10	Nov. 10	$2'{\cdot}25$,, ,,	3,500

Remarks.—1 and 2. These examples are noteworthy, because on this night, and this night only, there is substantial agreement between the values of the repulsion derived from the envelopes and from measures of points in the tail. The motion of the latter is exceptionally rapid on October 1 and 2, whereas this is nearly the lowest of the determinations from the envelopes. The envelopes are strongly hyperbolical. It may be noted (1) that the tail is filmy and resembles the envelopes in texture; (2) there are envelopes (dense, but with ill-defined edges) at a height of $5'$ from the nucleus towards the Sun—a height more than twice as great as on any other night.

3. A very good determination from four plates extending over $1\frac{3}{4}$ hours, the different points showing excellent agreement.

4. A rough determination.

5. Good.

6, 7, and 8. Much weaker.

9. Good determination, the results being very accordant.

10. Deduced solely from measures at latus rectum and vertex. Probably much less reliable than the other results.

§ 9. *Direct measurement of motion of matter projected towards the Sun.*—There is one isolated piece of evidence on this subject which it seems well to record. On Oct. 18 (the night when there was the best example of the rapid formation of an envelope, see page 447) there is a rather well-defined portion of matter which has been projected towards the Sun. The measures give

Plate.	Time.	x.	y.	$\frac{dx}{dt}$.	$\frac{dy}{dt}$.
	h m				
3173	7 58	+·090	+·244		
				·340	·028
3174	8 55	·416	·271		
				·339	·037
3175	9 50	·726	·305		

(Unit = 2′·5.)

The hourly motion in x is thus 0′·85.

Judging by the values of y, if $\frac{dy}{dt}$ is constant, the matter would appear to have left the nucleus at about 0^h; in that case, to satisfy the value of $\frac{dx}{dt}$, we must have $g = 0'\cdot22/\text{hr.}^2$ The evidence is rather uncertain; but as this would involve the matter reaching a height $x = -1'\cdot7$, g cannot very well be less. The corresponding value of μ is 420. This is greater (though not excessively greater) than the value of μ for the tail particles on that night.

§ 10. It is clear, then, that if we adopt the simple theory of the formation of envelopes, we are obliged to assume the action of repulsive forces 10 to 100 times as great as those which appear to act on the particles of the tail. Now there are two difficulties in the way of this. It is not clear that the envelopes can be sharply discriminated from the tail in the way this implies; as the envelope degenerates, it seems to mingle imperceptibly with the tail. Secondly, there are comets, such as Comet 1910 *a*, which seem to possess no tail, as distinct from the envelopes; probably among the comets which Bredichin examined were many of this type, and for these he must have actually measured the repulsive force on the envelopes and found it quite normal. Neither of these arguments is conclusive. It is quite possible that the envelope ultimately mingles with the tail, but any distinct point, whose motion can be directly measured, would almost necessarily be part of the tail proper, and not part of an envelope. Further, the envelopes of Comet Morehouse are apparently different in character from those to which Bredichin's method is applicable. Finally, there is considerable evidence that the solar repulsion ceases to act on the particles some time after their emission; this would account for the smaller values of μ obtained by those who study the remoter parts of the tail.

One assumption, however, is open to doubt, viz. that the velocity of projection is the same in all directions. The fact that the envelopes on the fore and rear sides of the comet do not generally correspond, or only correspond imperfectly, seems to show that the cone of emission is restricted, or not of uniform force. This renders the second method of calculating g somewhat uncertain, but it does not affect the first method. A decrease of V as a increases would render the envelope more elliptic, and would thus counterbalance the more hyperbolic form which a smaller value of g would produce. At the same time it is hard to believe that the approximately parabolic form, which is generally observed, is the result of such a chance counterbalancing of two opposing tendencies.* I think it is likely that V may vary with a sufficiently to require an appreciable reduction of some of the values of g found by the second method; for this reason I place more stress on the less precise evidence of the first method; but even as regards the second method it will be found that any attempt to work out the problem on the assumption that the repulsion has the value found from the study of the tail particles leads at once to difficulties, for it requires a duration of the envelope for a time much longer than is actually observed.

If we admit the values of g found by the two methods that I have explained, the initial velocity V is found to be of the order 10 to 100 kms. per second. It is, I suppose, impossible that such velocities could be produced in gases simply escaping under physical pressure, and we must look to some form of electrical or radio-active action to account for it. In forming such a theory there are three facts to be borne in mind: (1) at any instant the initial velocities of all particles emitted in any direction must be very nearly the same (otherwise a sharp linear envelope could not be formed); (2) the velocity of emission must decrease rather rapidly with the time; (3) the conditions must admit of two or more envelopes in different stages of collapse existing simultaneously.

§11. *Attractive or Repulsive force to the Nucleus.*—Another important question is, What modification would result if the nucleus either attracts or repels the particles? It is not possible to work out generally the effect of such forces; but I have traced by quadratures the paths of the particles in special cases, and have concluded that such forces would not help to explain the rapid formation of the envelope. A fairly large attractive force to the nucleus (following the Newtonian law) would modify the envelope in the following ways:

(1) It would make it more elliptical.

(2) It would *increase* (slightly) the time taken for the envelope formation to spread from the vertex to the latus rectum.

* The assumption that V decreases as a increases was made by Bredichin and Bond to account for the deviation from the parabolic form in the envelopes of certain comets; but the former considered it more probable (seeing that the truly parabolic form was the usual one) that in these cases the emission took place with constant velocity, but was confined to a small cone instead of filling the whole hemisphere. This latter explanation, however, only applies when the envelope is a mere boundary, and not when it is a line of great density.

(3) It would throw the matter more into the part near the vertex; *i.e.* instead of the envelope between the Sun and the focal plane being formed by matter ejected in a cone of angle 45°, it would be formed by a much larger cone.

(4) The matter would tend to hang for some time near the envelope instead of dissipating rapidly.

I have not examined a repulsive force, owing to the difficulty of assuming reasonable initial conditions. Owing to the smallness of the nucleus compared with the envelope, it seems impossible (under the inverse square law) to devise initial conditions under which the repulsive force would have very much effect.

It seems therefore useless to look for a solution of the difficulty in this direction.

§ 12. *Evidence for the Fountain-Theory of Envelopes.*—I do not see any other modification of the fountain-theory which can possibly explain the rapid formation of the envelopes. It remains, therefore, either to accept the large repulsive forces or to give up the fountain-theory, at least in the case of the sharp-line envelopes of Comet Morehouse. I will therefore conclude by examining the evidence on which the theory rests. In the first place, we can in many cases actually see the fan-shaped fountain of matter pointing towards the Sun, which the theory demands. Secondly, whereas any feature of the tail tends to dissipate and become less well-marked in later photographs, the envelopes start by being indistinct, and gradually become denser and more sharply defined. This, of course, is a natural result of the contraction of an envelope on the fountain-theory, and it is hard to see how it could be brought about in any other way. If it were not for this we might be tempted to regard them as *isochrones* (although they are not the right shape), or some other appearance that *consists permanently of the same material.* This is especially tempting, as in the motion of an envelope, $\frac{dx}{dt}$ (y constant) seems to agree very well with $\frac{dx}{dt}$ for tail matter at the same distance from the nucleus. But material must almost necessarily dissipate with time, instead of increasing in density, as the envelopes do.

§ 13. Nichols and Hull's suggestion (*Ap. J.*, xvii. p. 354) that the envelope forms at a height where condensation (caused by expansion and cooling of the emitted vapours) takes place, is worth very careful attention, considering the difficulties involved in the fountain-theory. But why should condensation take place almost instantaneously over a paraboloidal surface? Why, in particular, should the condensation only take place at a much greater height from the nucleus in directions away from the Sun than in directions towards the Sun? Once condensed, the envelope-material would be subject to light pressure, and would probably tend to collect more condensing matter; the theory would account admirably for the observed motion, changes of form, and increase of density of the envelope. But the initial formation, nearly instantaneously and in a parabolic form, presents an apparently insuperable difficulty.* The presence inside one another of a series of sharply-defined envelopes is also hard to understand.

§ 14. *Distribution of Density in the Paraboloid.*—The remarkable density and sharpness of the envelopes of Comet Morehouse is rather surprising when one has been accustomed to think of an envelope merely as a boundary. It is therefore desirable to show that it is a direct consequence of the fountain-theory, provided all the matter is expelled with the same velocity.

Take cylindrical coordinates (y, ϕ, x), x and y being measured as before. We have

$$\begin{aligned} x &= -\mathrm{V}t\cos\alpha + \tfrac{1}{2}gt^2 \\ y &= \mathrm{V}t\sin\alpha \\ \dot{x} &= -\mathrm{V}\cos\alpha + gt \\ \dot{y} &= \mathrm{V}\sin\alpha. \end{aligned}$$

Whence $v^2 = \dot{x}^2 + \dot{y}^2 = \mathrm{V}^2 + g^2t^2 - 2\mathrm{V}gt\cos\alpha$, where v is the velocity of the material; also $\dot{x}\sin\alpha + \dot{y}\cos\alpha = gt\sin\alpha$.

Now varying α and t,

$$dx = \dot{x}dt + \mathrm{V}t\sin\alpha d\alpha$$

$$dy = \dot{y}dt + \mathrm{V}t\cos\alpha d\alpha.$$

If we make dx, dy perpendicular to the velocity of the particles and in the plane xy, we must have

$$\dot{x}dx + \dot{y}dy = 0.$$

This leads to

$$(\mathrm{V}^2 + g^2t^2 - 2\mathrm{V}gt\cos\alpha)dt = -\mathrm{V}gt^2\sin\alpha d\alpha \quad . \quad (4)$$

Now $ds^2 = dx^2 + dy^2$

$$= (\dot{x}^2 + \dot{y}^2)dt^2 + \mathrm{V}^2t^2d\alpha^2 + 2\mathrm{V}t(\dot{x}\sin\alpha + \dot{y}\cos\alpha)dtd\alpha.$$

Therefore, using condition (4) for an element perpendicular to the direction of motion of the material, we find

$$ds^2 = \frac{\mathrm{V}^2t^2d\alpha^2}{(\mathrm{V}^2 + g^2t^2 - 2\mathrm{V}gt\cos\alpha)}(\mathrm{V} - gt\cos\alpha)^2.$$

Suppose that a steady state is reached and matter is emitted uniformly in all directions. Consider the matter emitted in an element of solid angle $\sin\alpha d\alpha d\phi$. It will flow in a tube of cross

* It is possible that a swarm of ions proceeding from the Sun and encountering the comet would, if repelled by the nucleus, part to either side; their paths would have as an inner envelope a roughly parabolic curve. The ions would serve as nuclei for condensation of the expelled gases.

section $ds.yd\phi$, and the quantity flowing across any cross section in unit time, namely,

$$\rho v ds.yd\phi,$$

will be proportional to $\sin a da d\phi$.

Therefore $\rho \mathrm{V} t da (\mathrm{V} \sim gt \cos a).\mathrm{V}t \sin a d\phi \propto \sin a da d\phi$.

Or the density ρ varies inversely as

$$t^2(\mathrm{V} \sim gt \cos a).$$

On the envelope $\mathrm{V} = gt \cos a$ and the density is infinite. As there are two tubes of flow passing any given point, corresponding to different solutions of equation (1), the total density is the sum of two parts, each given by the above expression.

Putting $u = \frac{1}{2}gt^2$, by a slight transformation the density may be written

$$\rho \propto \frac{1}{u_1 \left| (2a + x - u_1) \right|} + \frac{1}{u_2 \left| (2a + x - u_2) \right|}. \qquad (5)$$

where u_1, u_2 are roots of

$$u^2 - 2(x + 2a)u + x^2 + y^2 = 0.$$

In most cases, if not in all, the envelope-forming matter is only projected in the hemisphere towards the Sun. In that case, within the parabola K O K′ (see diagram, p. 444), formed by displacing the envelope a distance a along Ox, one of the tubes of flowing material will be missing, and ρ for such points consists of a single term. On this supposition I give the relative density of material in the paraboloid at different points in the cross sections $x = 0$ and $x = a$, that is, cross sections drawn through O and M respectively. η is the ordinate of the envelope for the cross section considered.

Distribution of Matter in the Paraboloid.

x \ $\frac{y}{\eta}$	0.	·2.	·4.	·6.	·707.	·8.	·9.	·95.	·99.	1·00.
0	∞	136·0	34·1	17·4	14·1	13·0	14·1	17·8	36·2	∞
a	0·61	0·62	0·69	0·84	6·00	5·77	6·53	8·27	17·2	∞

This table shows that the increase of density at the envelope is very sudden, so that a sharply-defined line may be expected. There is a minimum density at about $\frac{y}{\eta} = \cdot 8$, *i.e.* just within the envelope—a feature which is often noticeable on the photographs. Taking the cross section $x = a$, we see that for $y/\eta < \cdot 707$ there is very little material (owing to no ejection taking place in the hemisphere away from the Sun). This feature is very conspicuous in comets such as 1910 *a*, which show a divided "tail" with a dark space along the radius vector. There is no need to assume (as has been sometimes suggested) that the nucleus causes a sort of shadow; the figures show that the amount of matter projected towards the Sun and falling back into this space is in any case very insignificant. For $\frac{y}{\eta} > \cdot 707$ both the theoretical paths correspond to matter projected towards the Sun, so that there are now two streams contributing to the density. As y increases, the density diminishes at first, and then increases as the envelope is reached. There is thus a line of minimum density running within and near the envelope. The broken line U V represents (roughly) part of its course. This seems to explain Mr. Hinks' observation in the case of Comet 1910 *a*, that in each of the two symmetrical streams of envelope-matter the density was greatest at the edges.

It must be borne in mind that the appearance of the paraboloid, whose density is given above, will differ somewhat from the appearance of a plane section of it. It is not worth while working out the result mathematically, as the appearance must depend on the inclination of the axis to the line of vision, but the general considerations of the above paragraph will evidently hold good for the three-dimensional problem; and I think that the phenomenon of a minimum density occurring just within the envelope will be more prominent than in the two-dimensional case.

§ 15. *Conclusion.*—I feel personally reluctant to accept the very large repulsive forces indicated by the investigations of this paper, which seem to differentiate the envelope-particles from the tail-particles; on the other hand, I am not able to advocate any alternative explanation of the facts. The question clearly needs further examination, and I hope that the study of Halley's Comet may throw light on the matter. But it is necessary to recognise that Comet Morehouse was very abnormal in many respects, and it may be some time before a comet appears showing strictly similar phenomena.

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