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The 2:3:6 quasi-periodic oscillation structure in GRS 1915+105 and cubic subharmonics in the context of relativistic discoseismology

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ABSTRACT

We propose a simple toy model to explain the 2:3:6 quasi-periodic oscillation (QPO) structure in GRS 1915+105 and, more generally, the 2:3 QPO structure in XTE J1550−564, GRO J1655−40 and H1743−322. The model exploits the onset of subharmonics in the context of discoseismology. We suggest that the observed frequencies may be the consequence of a resonance between a fundamental g mode and an unobservable p wave. The results include the prediction that, as better data become available, a QPO with a frequency of twice the higher twin frequency and a large quality factor will be observed in twin peak sources, as it might already have been observed in the especially active GRS 1915+105.

Key words: accretion, accretion discs – black hole physics – X-rays: binaries.

1 INTRODUCTION

In spite of active research, the remarkable structure in the power spectra of several X-ray binaries remains a major puzzle for over a decade now. Each one of the four black hole sources that show more than one high frequency (40–450 Hz) quasi-periodic oscillation (HFQPO) exhibits two of them in the 'twin peaks' 2:3 ratio (see Table 1).

An understanding of HFQPOs may allow us to obtain important information about the corresponding black hole's mass and spin, and the behaviour of inner-disc accretion flows.

The physics of HFQPOs is not completely understood. Since the observed 2:3 ratio suggests the presence of non-linear physics, resonant models have been proposed. Thus, starting with the pioneering work of Kluźniak & Abramowicz (2001), explanations of the observed ratio have been sought by means of a parametric resonance among the dynamical (orbital and epicyclic) frequencies (for more recent developments, see e.g. Török et al. 2006; Stuchlík, Kotrlová & Török 2013). Moreover, Kato (2008) considers long-wavelength disc deformations which couple non-linearly to disc oscillations. A detailed discussion of models can be found in Török et al. (2011).

Even though a considerable amount of research has thus already focused on non-linear resonances, there is still room for further exploitation of subharmonics, especially given the fact that other methods yield quasi-periodic oscillation (QPO) frequencies which are too high to match observations (Török et al. 2011).

The main objective of this paper is thus to explore the idea of subharmonics beyond previous efforts. Subharmonics, which have already been identified in stars (e.g. in RV Tauri-type variables; Pollard et al. 1996), have also been theoretically discussed in the context of accretion discs by Kluźniak, Abramowicz & Lee (2004) and Rebusco et al. (2012), although both papers avoid details of disc models.

In this paper, we would like to develop this discussion by exploring the inclusion of cubic (in addition to quadratic) subharmonics in the context of relativistic discoseismology, the formalism of normal mode oscillations of thin accretion discs (for a review, see Wagoner 2008).

According to discoseismology, the observed oscillations in the outgoing X-ray radiation of systems such as GRS 1915+105 are due to normal modes of adiabatic hydrodynamic perturbations. These modes are the result of gravitational and pressure restoring forces in a geometrically thin, optically thick accretion disc in the steep power-law state.

This interpretation is not only corresponded observationally by narrow peaks in the power spectral density, but some of these modes have been observed in hydrodynamic simulations as well (Reynolds & Miller 2009).

Assuming that this formalism is correct, we may use it to build an exploratory study of non-linear effects. If one thinks of discoseismic modes as harmonic oscillators, one might be able to model non-linearities in the fluid equations as non-linear terms added to a simple oscillatory system.

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Table 1. The frequencies of 2:3 QPO twin peaks in microquasars and the corresponding black hole spin. The values for the non-dimensional black hole angular momentum parameter $a \equiv cJ/GM^2$ are averages of the different methods; numbers in parentheses express the standard deviation when there is more than one method. References: (1) Wagoner (2012; and references therein); (2) Török et al. (2011); (3) Remillard et al. (2003a); (4) McClintock & Remillard (2006); (5) Belloni et al. (2006); (6) Remillard et al. (2003b); (7) Remillard et al. (2002); (8) Remillard et al. (1999); (9) Strohmayer (2001); (10) Remillard et al. (2006); (11) Homan et al. (2005).

Source	Frequencies (Hz)	Black hole spin (a)	References
GRS 1915+105	113±5 168±5	0.79 (19)	1, 2, 3, 4, 5, 6
XTE J1550−564	184±5 276±2	0.60 (28)	1, 7
GRO J1655−40	300±9 450±5	0.86 (14)	1, 7, 8, 9
H1743−322	165±6 241±3	0.20	1, 10, 11

When devising a toy model for the disc's complicated dynamics, our aim was to propose the simplest mathematical expression which has solutions that include quadratic and cubic subharmonics in a compact and observationally productive way. It turns out that this can be achieved with a *single* one-dimensional non-linear driven oscillator, described by the following equation:

$$\ddot{x} + \omega_0^2 x - \varepsilon x^2 - \delta x^3 = B \cos \omega t \quad (1)$$

(Landau & Lifshitz 1994). More complicated ordinary differential equations (ODEs) or a system of coupled oscillators may be more realistic but at the expense of being less insightful. Equation (1) captures the essential properties without unnecessarily obscuring the discussion. As we discuss below, this oscillator features subharmonics that appear in a bifurcative way and which are thus not obtainable by analytic continuation of linearized perturbation theory.

The determination of the physical content of the terms in (1) constitutes the core of this paper, and this is the matter of Section 3. One would like to know, for example, what fraction of the hydrodynamic oscillation energy lies in non-linear interactions, and how would this information be related to the peaks' properties in the power spectra. Before that, however, we will describe the model in detail in Section 2, while the main discussion, including numerical results and predictions, occupies Section 4.

2 THE MODEL

2.1 Background

Modelling of non-linear oscillations in rotating stars has been developed by Dyson & Schutz (1979), Schutz (1980a,b), Kumar & Goldreich (1989), Wu & Goldreich (2001), Schenk et al. (2002), and Arras et al. (2003). In this type of formalism, perturbation theory is used to find the corrections to the linear regime in the form of non-linear couplings between (otherwise uncoupled) normal modes. This type of formalism was applied by Horák (2008) to the case of slender tori as a rough approximation to the oscillating region in an accretion disc.

One can thus calculate, in stars and discs, mode-coupling coefficients between three or more modes. These coefficients provide important information about the system (assuming non-linearities are mild), such as selection rules for the participating modes and relative strengths of the couplings, in addition to slight changes in mode frequency ('detuning') and stability considerations.

However, this approach has limitations and fails once the amplitude of the oscillations becomes large enough (even while still being in the perturbative regime). Then, matters can become intractable as the appearance, via pitchfork bifurcation, of new forms of oscillation renders analytical approaches useless.

Vakakis (1997) illustrates the concepts by means of a toy system of two masses connected by non-linear springs. Even though the system has only two degrees of freedom, it can develop (under appropriate conditions) three forms of oscillation, which he dubs *non-linear normal modes*.

2.2 Physical interpretation

Our toy model is a device that incorporates succinctly the additional aforementioned modes, and it is thus more than a simplified visual depiction of the system. We shall assume that equation (1) represents the dynamics of the QPOs *after* it has reached a stationary state, the (toy) variable x being a measurement of fluid displacement. Let us further assume that ε and δ are small enough that perturbative considerations make sense.

Assume that the oscillator described by the first two terms on the left-hand side of equation (1), $\ddot{x} + \omega_0^2 x$, represents a discoseismic fundamental (axisymmetric) g mode (an inertial-gravitational oscillation), so we set ω_0 to be the g-mode frequency. Let us call this frequency $\nu_2 \equiv \omega_0$. Throughout this paper, we use the convenient notation $\nu_n \equiv (n/2)\omega_0$.

Assume further that ω represents a higher frequency oscillation, associated with discoseismic axisymmetric p waves (inertial-acoustic oscillations). Let us set $\omega = 3\omega_0$ and thus call it $\nu_6 = \omega$. As shown in Fig. 1, the conditions for the existence of axisymmetric g modes and p waves are that the discoseismic eigenfrequency σ be smaller and greater, respectively, than the radial epicyclic frequency κ.

In the absence of the non-linear terms, the spectrum of the system described by equation (1) would contain ω_0 and ω, i.e. ν_2 and ν_6, and nothing else. The presence of the term εx^2 generates a subharmonic of value $(3/2)\omega_0$ (i.e. half of ω), which we call ν_3, while the term δx^3 is necessary for the resonance between the ω_0 and the ω oscillators. (For a theoretical discussion of subharmonics, see e.g. Jordan & Smith 2007; for experimental results, see e.g. Linsay 1981.)

The frequencies ν_2 and ν_3 would constitute the observed lower and higher QPO twin frequency pair. But note that our model requires as well the existence of a ν_6 QPO frequency (see Fig. 1). Even though there is one notable instance of this 2:3:6 structure in GRS 1915+105 (Remillard et al. 2003b; McClintock & Remillard 2006), this is rare and presumably the ν_6 is in general either a transient phenomenon or too weak to be observed, as non-linearities can render subharmonics that are larger than the driving source.

We emphasize that equation (1) represents an *effective* equation that describes the system in a formal way. The ω oscillator should not be regarded as an 'external' oscillator in a physical sense. Physically, it would be more natural to think about the ω_0 (g-mode) oscillator as the one driving the motion. Thus, even though our model was conceived primarily to deal with the stationary state of the disc, and not with its evolution, the following sequential scenario suggests itself. Once a g-mode (ν_2) and the p-wave (ν_6) oscillations are established via the cubic resonance, then the quadratic resonance generates a ν_3 frequency. In this way, the observed frequencies may be a consequence of a resonance between fundamental g modes and

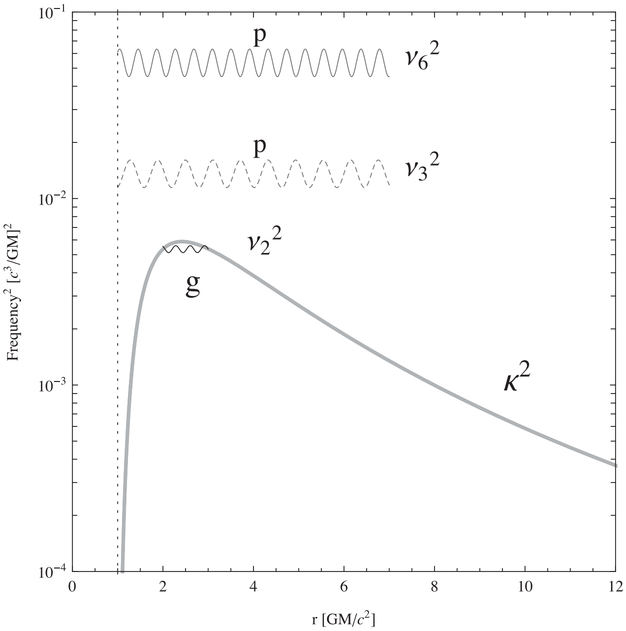

Figure 1. The location of axisymmetric discoseismic oscillation modes (wavy lines) and values of the square of their frequencies are plotted as a function of the radius. The ν_2 frequency corresponds to the fundamental g mode, while ν_3 and ν_6 are p waves. Also shown are the square of κ (the radial epicyclic frequency) for the case $a = 1$, and the position of the inner edge of the disc (dashed vertical line). The frequencies ν_2, ν_3 and ν_6 are in a 2:3:6 relationship, and thus ν_2 and ν_3 produce according to our model the twin peak pair.

unobservable p waves. (A more comprehensive approach, involving all oscillators simultaneously, is described in Section 4.2.)

2.3 Why linear discoseismology is insufficient

A natural question to ask is just why higher frequency discoseismic g modes cannot be used instead of p waves as the ω resonator above. After all, g modes have frequencies in the right range, and are the most robust modes, as they exist in the hottest part of the disc and lie away from the uncertain physics of the inner boundary (Perez et al. 1997). Modes with different angular mode number m (such that oscillations are proportional to $e^{im\phi}$) would seem to be good candidates.

For non-rotating black holes ($a = 0$), the frequencies corresponding to angular mode numbers $m = 0, 1, 2$ are in a $1 : 3.5 : 6.4$ relation. For the $a = 0.5$ and $a = 0.998$ cases, the relations are $1 : 3.7 : 6.8$ and $1 : 5.7 : 11.3$, respectively. (The dependence of the frequencies on the radial and vertical mode numbers is very weak, changes amounting to a few per cent or less.)

Thus, there are no modes close to three times the fundamental frequency. In addition, the absence of negative m g modes causes the relevant non-linear couplings to vanish due to selection rules of the form $\Sigma\, m_i = 0$ (Arras et al. 2003).

3 PHYSICAL MEANING OF THE PARAMETERS

In order to further exploit the model, we need to make a connection to more realistic (though not subharmonic producing) models. In particular, we would like to obtain values for ε and δ in equation (1) in terms of physical variables.

We will proceed thus to match our model with the values obtained by Horák (2008) for the coupling coefficients of oscillations in slender tori around of black holes. This will allow us to obtain order-of-magnitude values for our parameters.

We begin by reexpressing (1) in the following form: (which is the one used by Horák 2008)

$$\dot{y} + i\omega_0 y = i\omega_0 (E y^2 + \Delta y^3), \tag{2}$$

in terms of the non-dimensional quantities

$$y \equiv \frac{x}{A}, \quad E \equiv \frac{\varepsilon A}{3\omega_0^2}, \quad \Delta \equiv \frac{\delta A^2}{4\omega_0^2}, \tag{3}$$

where A is the amplitude of the (toy) oscillation. We have taken for now $B = 0$, as the interesting part is in the non-linear terms anyway, and higher order terms $\propto \varepsilon^2$ have been dropped.

In this form, $y \sim 1$, while the conditions for the validity of the perturbative approach now read $|E| < 1$, $|\Delta| < 1$. The quantities $|E|$ and $|\Delta|$ have a simple physical meaning: they represent the ratio of the non-linear interaction energy to the energy of the linear mode (for the quadratic and cubic terms, respectively).

Equations (5), (6) and (8) in Horák (2008) for the fluid displacement $\boldsymbol{\xi}$ and the coupling coefficients κ and b are: (indices refer to modes)

$$\boldsymbol{\xi}(t, \boldsymbol{x}) = \sum c_A^*(t)\, \boldsymbol{\xi}_A^*(\boldsymbol{x}), \tag{4}$$

$$\dot{c}_A + i\omega_A c_A = i b_A^{-1} \mathcal{F}_A^*, \tag{5}$$

$$\mathcal{F}_A = \sum \kappa_{ABC}\, c_B c_C + \sum \kappa_{ABCD}\, c_B c_C c_D + \cdots. \tag{6}$$

In this order-of-magnitude approach, we drop the indices in c_A and b_A. Since $|c| \sim 1$ for mild non-linearities, we can now easily compare equations (2) and (5), and identify c with y.

This comparison can be readily performed once we have an order-of-magnitude estimate for the (pressure- and gravity-dominated) couplings:

$$\kappa_{ABC}^{(p)} \sim \kappa_{ABC}^{(g)}(R/h) \sim b\,\Omega(\xi/h), \tag{7}$$

where Ω, ξ, h and R stand, respectively, for the dynamical inverse time-scale and the displacement, disc thickness and disc radius length-scales. We have used $\xi/h \sim \Delta\rho/\rho$, the fractional mass density, and we have assumed that all the components of ξ have the same order-of-magnitude value.

4 DISCUSSION

4.1 General results

The considerations of the previous section allow us to conclude, taking $\omega_0 \sim \Omega$, that $|E| \sim \xi/h$ and $|\Delta| \sim E^2$.

Since we have that $|E|$, $|\Delta| < 1$ (our perturbative condition) and that Nowak & Wagoner (1993) found that $\xi/h \sim 1$, we may conclude the following. $|E|$ has to be smaller than 1 yet not much smaller than 1 (meaning that our system is barely perturbative), while $|\Delta|$ has to be smaller than $|E|$. (It will be reassuring to obtain solutions with values consistent with this reasoning below; see Table 2.)

Since $|\Delta| < |E|$, it follows from (2) that the effects related to the quadratic term will be somewhat stronger than those for the cubic one. This means that the QPO peak amplitude should be higher for ν_3 than for ν_2, which is exactly what is observed. Remarkably, this feature holds for *all* of the four sources in Table 1. (For observational reviews, see McClintock & Remillard 2006, p. 157; Belloni, Sanna

& Méndez 2012.) The reasoning behind this statement becomes clearer if one thinks of the non-linear terms as driving forces.

Moreover, if one thinks of (2) as a spring (left-hand side) with driving forces (right-hand side), and if one further assumes there is a damping term $2\beta\dot{x}$, with a quality factor $Q \equiv \omega_0/2\beta$, then there will be, according to our model, an inverse relationship between the ratio $r \equiv$ amplitude of ν_3/amplitude of ν_2 (by 'amplitude' we mean the standard PSD amplitude) and Q. This feature is also observed. The variables r and Q show a correlation of -0.49 when the value of Q for the lower twin frequency is used. (This correlation and the one in the next paragraph have been calculated directly from the data in the references listed in Table 1.)

The plausibility of the model is further supported by the following observation. There exists a very strong correlation ($+0.91$) between the amplitude of the lower twin frequency and its Q, while there is no significant correlation between the amplitude of the higher twin frequency and its Q. This suggest that the two QPO twin frequencies arise from different physical mechanisms, and that the lower twin frequency might play a more primary role in the dynamics (e.g. that of being the driver of the QPO system), since it has a property that is degraded in the other QPO frequency peak. Such an interpretation is consistent with the identification of the toy spring frequency with the robust discoseismic fundamental g mode.

4.2 Parameter determination

In order to obtain further results from our toy model, it is useful now to consider all of the relevant frequencies together. Thus, we substitute

$$x(t) = C\cos(\omega t) + D\cos(\omega t/2) + F\cos(\omega t/3) + K \quad (8)$$

in (1). This procedure yields of course several cross terms in addition to the purely quadratic and cubic subharmonic contributions. By grouping the $\cos(\omega t)$, $\cos(\omega t/2)$ and $\cos(\omega t/3)$ terms, it is thus possible to obtain a system of three coupled cubic equations for D/C, F/C and K/C in terms of the parameters E, Δ and $B/(C\omega^2)$, all six quantities being non-dimensional.

A numerical solution produces values for $(D/F)^2$, i.e. the quantity we called r in the previous section. We assume that the non-linearities cause locking of the phases of the terms in (8), as it is usual for non-linear systems (Pikovsky, Rosenblum & Kurths 2003). There are only five different solutions for $(D/F)^2$ when scanning the parameter space with the 65 000-point grid given by: $0.1 < |E| < 0.5$ (step = 0.05), $0.33\,E^2 < |\Delta| < 3\,E^2$ (step = 0.33), $0.1 < |B/(8C\omega^2)| < 10$ (step = 0.1).

As Table 2 shows, most of the solutions have $(D/F)^2 > 1$. The fourth solution is the closest to the observational value of ≈ 2 (as can be appreciated directly from the plots in the references of Table 1). As expected, $|E|$ and $|\Delta|$ satisfy the properties described at the beginning of Section 4.1. The values of $B/(C\omega^2)$ are close to -8, its value for the $\varepsilon = \delta = 0$ case.

Table 2. The solution values of $(D/F)^2$ obtained numerically from equation (8), with values of the parameters. The quantity $(D/F)^2$ corresponds to the higher twin/lower twin PSD amplitude ratio. $|E|$ is bigger than $|\Delta|$, showing dominance of the quadratic subharmonic over the cubic one.

$(D/F)^2$	E	Δ	$B/(C\omega^2)$
21	±0.5	0.083	−7.2
2.4	±0.5	0.083	−7.2
4.6	±0.5	−0.09	−8
1.5	±0.45	−0.073	−8
0.16	±0.4	−0.058	−8

Thus, even in the toy model approximation, these numerical results yield a sensible outcome and have the expected parameter values. They imply that about half of the oscillation energy lies in non-linear interactions, especially those associated with the quadratic subharmonic.

4.3 Predictions

In the first place, the model predicts that, as better data and/or analysis become available, a QPO frequency equal to ν_6 will be revealed, as it already appears to be present in GRS 1915+105. Furthermore, given that ν_6 presumably forms before, and is the cause of, its subharmonic ν_3, it is expectable that ν_6 will be less degraded, i.e. have a larger value of Q, than ν_3 (as it already does for the case of GRS 1915+105).

Secondly, QPO combination frequencies with values $\nu_3 \pm \nu_2$, i.e. ν_1 and ν_5 in our notation, may be observed, but *only* when ν_2 and ν_3 are present at the same time. Even though the 1:2:3 harmonic observations from XTE J1550−564 are favourable in this respect, there are currently not enough data to state anything conclusive in this regard.

4.4 Difficulties and future work

A challenge of the model concerns explaining the missing ν_4, given that the arguments applied to ν_6 apply to ν_4 as well, i.e. why is there not a resonance between ν_2 and ν_4 via the term εx^2?

Table 3 summarizes the information of the different discoseismic p waves. The size of each wave is estimated by the discoseismic radial wavelength (Ortega-Rodríguez, Silbergleit & Wagoner 2008).

The physical behaviour at the inner disc boundary is probably the least understood aspect of the whole accretion disc system, given the sudden domination of magnetic field and coronal effects there (Hawley & Krolik 2001). In particular, it is not known whether there is wave leaking or reflection, let alone phase change.

Assume, however, and for the sake of the present discussion, that there is enough of an impedance mismatch at the inner boundary that the p-wave oscillations bounce back without changing phase (assuming a free boundary condition). Standing waves could then in principle be created for the waves in Table 3, in what we may call 'discoseismic semi-modes' (since there is only one boundary, the inner one). Alternatively, it may be the case that the non-linear character of the system confines the oscillations away from the boundary as explained in Vakakis (1997) for certain mechanical systems.

Table 3. The properties of theoretical p-wave oscillations relevant to the discussion. Wavelengths (λ) are obtained from the discoseismic formula $\lambda^2 = h^2(1 - \kappa^2/\sigma^2)^{-1}$, where h and κ refer to the disc's thickness and the radial epicyclic frequency, respectively. Frequencies ν_3, ν_4 and ν_6 are in a 3:4:6 relation.

Frequency (σ)	Size	Observational status
ν_6	$\lambda_6 = 1.06\,h$	Only observed in GRS 1915+105
ν_4	$\lambda_4 = 1.15\,h$	Not observed
ν_3	$\lambda_3 = 1.34\,h$	Observed in all four sources in Table 1

In either case, since the distance $r(\kappa_{\max}) - r_i$ (the radius at which the radial epicyclic frequency is maximum minus the radius of the inner disc boundary) is not much larger than h, especially for fast spinning black holes (Perez et al. 1997), the discoseismic p waves will only travel a few wavelengths before bouncing back and returning.

This sets up a scenario in which ν_6 and ν_3 can build up but ν_4 does not: if the round trip distance, which is given by twice $r(\kappa_{\max}) - r_i$, equals e.g. $5 \times \lambda_6 \approx 4 \times \lambda_3 \approx 4.6 \times \lambda_4$, then there would be true ν_6 and ν_3 semi-modes, while the necessary conditions for ν_4 to exist will not be met, as it would be out of phase.

If this model is on the right track, and thus a sizable fraction of the energy resides in non-linear interactions, then a more careful study of subharmonic dynamics is in order. One might develop a full (coupled) model, or even investigate the possibility of solitonic confinement of the oscillations.

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