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Solar wind and the motion of dust grains

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ABSTRACT

In this paper, we investigate the action of solar wind on an arbitrarily shaped interplanetary dust particle. The final relativistically covariant equation of motion of the particle also contains the change of the particle's mass. The non-radial solar wind velocity vector is also included. The covariant equation of motion reduces to the Poynting–Robertson effect in the limiting case when a spherical particle is treated, when the speed of the incident solar wind corpuscles tends to the speed of light and when the corpuscles spread radially from the Sun. The results of quantum mechanics have to be incorporated into the physical considerations, in order to obtain the limiting case.

If the solar wind affects the motion of a spherical interplanetary dust particle, then $\boldsymbol{p'}_{\rm out} = (1 - \sigma'_{\rm pr}/\sigma'_{\rm tot})\boldsymbol{p'}_{\rm in}$. Here, $\boldsymbol{p'}_{\rm in}$ and $\boldsymbol{p'}_{\rm out}$ are the incoming and outgoing radiation momenta (per unit time), respectively, measured in the proper frame of reference of the particle, and $\sigma'_{\rm pr}$ and $\sigma'_{\rm tot}$ are the solar wind pressure and the total scattering cross-sections, respectively.

An analytical solution of the derived equation of motion yields a qualitative behaviour consistent with numerical calculations. This also holds if we consider a decrease of the particle's mass. Using numerical integration of the derived equation of motion, we confirm our analytical result that the non-radial solar wind (with a constant value of angle between the radial direction and the direction of the solar wind velocity) causes outspiralling of the dust particle from the Sun for large values of the particle's semimajor axis. The non-radial solar wind also increases the time the particle spirals towards the Sun. If we consider the periodical variability of the solar wind with the solar cycle, then there are resonances between the particle's orbital period and the period of the solar cycle.

Key words: celestial mechanics – solar wind – interplanetary medium – zodiacal dust.

1 INTRODUCTION

For many decades, the Poynting–Robertson (P–R) effect has been used to model the orbital evolution of dust grains under the action of the electromagnetic radiation (of the central star; e.g. Poynting 1903; Robertson 1937; Wyatt & Whipple 1950; Dohnanyi 1978; Kapišinský 1984; Jackson & Zook 1989; Leinert & Grün 1990; Gustafson 1994; Dermott et al. 1994; Reach et al. 1995). It has been found that the solar wind operates in a similar way. The action of the solar wind on the motion of an interplanetary dust particle (IDP) has been discussed, in a heuristic way, for example, by Whipple (1955). He also mentioned the results of laboratory experiments: the intense bombardment of a material by energetic corpuscles destroys the material, an effect known as 'sputtering'. Current opinion is that the solar wind has two different effects: (i) the motion of a dust particle is influenced by the incident solar wind; (ii) the corpuscular sputtering decreases the mass of the particle (e.g. Whipple 1955; Dohnanyi 1978; Kapišinský 1984; Leinert & Grün 1990). There have been attempts to better understand the physics of the action of the solar wind on the motion of a dust particle. A first attempt was made by Robertson & Noonan (1968, pp. 122-123), who formulated the relativistically covariant equation of motion of a particle under the action of the solar wind. However, their result does not reveal any destruction of the particle. A more realistic view has been presented by Klačka & Saniga (1993), who also suggested a space–time formulation of the problem. As a result, corpuscular sputtering is an indispensable part of the equation of motion for the action of solar wind on an IDP. A relativistically covariant form of the equation of motion with corpuscular sputtering has not yet been

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presented in the literature. In this paper, we generalize the results of Klačka & Saniga (1993) by deriving a relativistically covariant equation of motion with corpuscular sputtering included. Using the results of Klačka & Saniga (1993), Klačka (1994) derived the equation of motion to the first order of v/u, where v is the speed of the dust grain with respect to the Sun and u is the speed of the solar wind with respect to the Sun. In general, the solar wind velocity vector can be non-radial. The non-radial component of the solar wind velocity vector has also been considered in the equation of motion to the first order of v/u (Klačka 1994). Klačka et al. (2008) and Pástor et al. (2009a) used the acceleration caused by the non-radial solar wind to the second order in v/u. The acceleration in Klačka et al. (2008) and Pástor et al. (2009a) was used without one term of the second order in v/u, as follows from the covariant formulation presented in this paper. We explicitly present the complete acceleration caused by the non-radial solar wind to the second order in v/u. Klačka et al. (2008) used the acceleration from the P–R effect and the non-radial solar wind in order to obtain the properties of the orbital evolutions of dust particles near to (or captured in) a mean-motion resonance with a planet. Pástor et al. (2009a) analysed in detail all possibilities of the secular evolutions of eccentricity in a mean-motion resonance with a planet under the action of the P–R effect and the non-radial solar wind.

In this paper, we present a space–time formulation of the action of the solar wind on an arbitrarily shaped IDP. We derive the equation of motion in a relativistically covariant form. Moreover, in order to be physically correct, the results of quantum theory are also taken into account. The results of the paper are consistent with the results of Klačka (2008a,b) for electromagnetic radiation. Our theoretical derivations hold for any solar wind velocity vector, and the result can easily be applied to other stars with stellar winds.

We present the application of the derived equation of motion to a spherical IDP in the form of the orbital evolution of the particle. While submicrometre dust particles are driven mainly by the Lorentz force (the motion of charged particles in the interplanetary magnetic field; Dohnanyi 1978, Leinert & Grün 1990, Dermott et al. 2001), collisions among particles are important for particles that have radii larger than ≈100 μm (Grün et al. 1985; Dermott et al. 2001). We deal with the orbital evolution of a μm-sized spherical IDP, when the effects of solar gravity, solar electromagnetic radiation and solar wind (solar corpuscular radiation) are relevant. The radial solar wind is usually used. However, the newest observations (Bruno et al. 2003) show that the velocity vector of solar wind corpuscles is non-radial and that the angle between the velocity vector and the radial direction is practically independent of heliocentric distance. We compare the orbital evolution of a spherical IDP for the standard approach to the time-independent radial solar wind and for the more real solar wind model. We also take into account the effect of the decrease of the mass of the IDP.

In Section 2, we derive the relativistically covariant equation of motion of an arbitrarily shaped particle under the action of solar wind (including the non-radial component of the solar wind velocity). In Section 3, we summarize important equations for the P–R effect. In Section 4, we give the equation of motion of the spherical particle under the action of solar corpuscular and electromagnetic radiation and solar gravity. In Section 5, we deal with the secular evolution of the particle's orbital elements under the action of solar radiation (electromagnetic and corpuscular – solar wind), in an analytical way. In Section 6, we provide a detailed treatment of the numerical results, and we compare the results for the conventional time-independent radial solar wind with those obtained for the more real solar wind model.

2 EQUATION OF MOTION: SOLAR WIND EFFECT

In this section, we derive the relativistically covariant equation of motion of an IDP. Our derivation enables us to understand the physics of the action of the solar wind on the motion of a dust particle (compared with the heuristic explanation from Whipple 1955 and the space–time formulations presented by Robertson & Noonan 1968, pp. 122–123 and Klačka & Saniga 1993).

We show that corpuscular sputtering is an indispensable part of the action of the solar wind on the IDP, so sputtering cannot be considered as an another effect of the solar wind. Moreover, the non-radial solar wind velocity vector can easily be incorporated into the final equation of motion. Finally, the covariant formulation yields the P–R effect in the limiting case when the speed of the solar wind corpuscles tends to the speed of light. The limiting case is fulfilled assuming that the total cross-section of the interaction between the solar wind corpuscles and the IDP is given by the results of quantum theory and not by classical non-quantum physics.

We also present the results with an accuracy of the order of $(v/u)^2$, where v is the speed of the particle with respect to the Sun and u is the solar wind speed with respect to the Sun. These results are used in the practical modelling of the orbital evolution of interplanetary dust grains in Sections 4 and 5.

2.1 Incident radiation

Let us introduce two inertial reference frames. The first is the proper reference frame of a particle moving with velocity $\boldsymbol{v}$ around the Sun. The particle is at rest in its proper frame of reference. The quantities measured in this frame are denoted by a prime. The second frame is associated with the Sun. This frame is the 'stationary reference frame'.

We suppose that all corpuscles of the solar wind are of the same mass m_1 and of the same velocity $\boldsymbol{u}$ (or $\boldsymbol{u}'$ in the proper reference frame of the particle). Thus, for the four-momentum of each of the corpuscles in the proper reference frame of the IDP, we can write

$$p'^{\mu}_1 = \left(\frac{E'_1}{c}; \boldsymbol{p}'_1 \right) = m_1 \left[\gamma \left(u' \right) c; \gamma \left(u' \right) \boldsymbol{u}' \right] . \tag{1}$$

Here, c is the speed of light and

$$\gamma \left(u' \right) = \frac{1}{\sqrt{1 - u'^2/c^2}} \tag{2}$$

is the Lorentz factor. Similarly, in the stationary reference frame, each of the corpuscles has the following four-momentum

$$p^{\mu}_1 = \left(\frac{E_1}{c}; \boldsymbol{p}_1 \right) = m_1 \left[\gamma \left(u \right) c; \gamma \left(u \right) \boldsymbol{u} \right] . \tag{3}$$

If a beam of such solar wind corpuscles hits the dust grain, then the energy and momentum incident on the particle per unit time in its proper frame are

$$E'_{\rm in} = \sigma'_{\rm tot} n' u' E'_1 \qquad \text{and} \qquad \boldsymbol{p}'_{\rm in} = \sigma'_{\rm tot} n' u' \boldsymbol{p}'_1 , \tag{4}$$

respectively. Here, n' is the concentration of the solar wind corpuscles and $\sigma'_{\rm tot}$ is the total scattering cross-section of the IDP. Using equation (1), we can rewrite the equations of equation (4) in the form of the incident four-momentum per unit time

$$p'^{\mu}_{\rm in} = \sigma'_{\rm tot} n' u' \left(\frac{E'_1}{c}; \boldsymbol{p}'_1 \right) = \frac{1}{c} \sigma'_{\rm tot} n' u' E'_1 \left(1; \frac{\boldsymbol{u}'}{c} \right) . \tag{5}$$

If we introduce the flux density of the incident energy (the energy flow per unit area perpendicular to the beam of solar wind corpuscles per unit time)

$$S' = n'u'E'_1, \tag{6}$$

then equation (5) can be rewritten as follows:

$$p'^{\mu}_{\text{in}} = \left(\frac{E'_{\text{in}}}{c}; \boldsymbol{p'}_{\text{in}}\right),$$
$$p'^{\mu}_{\text{in}} = \frac{1}{c} S' \sigma'_{\text{tot}} \left(1; \frac{\boldsymbol{u'}}{c}\right). \tag{7}$$

With a four-vector $B'^{\mu} = (B'^0; \boldsymbol{B'})$ in the proper reference frame, the components of the four-vector in the stationary reference frame are given by the following generalized special Lorentz transformation

$$B^0 = \gamma(v)\left(B'^0 + \frac{\boldsymbol{v}\cdot\boldsymbol{B'}}{c}\right),$$
$$\boldsymbol{B} = \boldsymbol{B'} + \left\{[\gamma(v) - 1]\frac{\boldsymbol{v}\cdot\boldsymbol{B'}}{v^2} + \frac{\gamma(v)}{c}B'^0\right\}\boldsymbol{v}, \tag{8}$$

or inverse

$$B'^0 = \gamma(v)\left(B^0 - \frac{\boldsymbol{v}\cdot\boldsymbol{B}}{c}\right),$$
$$\boldsymbol{B'} = \boldsymbol{B} + \left\{[\gamma(v) - 1]\frac{\boldsymbol{v}\cdot\boldsymbol{B}}{v^2} - \frac{\gamma(v)}{c}B^0\right\}\boldsymbol{v}. \tag{9}$$

Here, $\boldsymbol{v}$ is the velocity of the proper reference frame with respect to the stationary reference frame, and $v = |\boldsymbol{v}|$.

Now, using equations (7) and (8), we obtain

$$p^0_{\text{in}} = \frac{1}{c} S'\sigma'_{\text{tot}}\gamma(v)\left(1 + \frac{\boldsymbol{v}\cdot\boldsymbol{u'}}{c^2}\right),$$

$$\boldsymbol{p}_{\text{in}} = \frac{1}{c^2} S'\sigma'_{\text{tot}}\left[\boldsymbol{u'} + \left\{[\gamma(v) - 1]\frac{\boldsymbol{v}\cdot\boldsymbol{u'}}{v^2} + \gamma(v)\right\}\boldsymbol{v}\right]. \tag{10}$$

We have to express the primed quantities (except for σ'_{tot}) on the right-hand sides of the equations in equation (10), i.e. $S' = n'u'E'_1$ and $\boldsymbol{u'}$, using the unprimed quantities measured in the stationary reference frame of the Sun. We obtain the energy E'_1 from the Lorentz transformation of p^{μ}_1 to the proper reference frame of the IDP as

$$E'_1 = \gamma(v)\left(E_1 - \boldsymbol{v}\cdot\boldsymbol{p}_1\right)$$
$$= \gamma(v)\left(1 - \frac{\boldsymbol{v}\cdot\boldsymbol{u}}{c^2}\right)E_1 = \omega E_1, \tag{11}$$

where we have defined the quantity

$$\omega \equiv \gamma(v)\left(1 - \frac{\boldsymbol{v}\cdot\boldsymbol{u}}{c^2}\right). \tag{12}$$

We obtain the other quantities from the transformation of the four-vector of the current density $j^{\mu} = (nc; n\boldsymbol{u})$ to the corresponding four-vector $j'^{\mu} = (n'c; n'\boldsymbol{u'})$. The transformation yields

$$n' = \omega n \qquad \text{and} \qquad \boldsymbol{u'} = \frac{1}{\omega}\boldsymbol{\alpha}, \tag{13}$$

where the vector

$$\boldsymbol{\alpha} \equiv \boldsymbol{u} + \left\{[\gamma(v) - 1]\frac{\boldsymbol{v}\cdot\boldsymbol{u}}{v^2} - \gamma(v)\right\}\boldsymbol{v}, \tag{14}$$

has the magnitude

$$\alpha = \left[u^2 + \gamma^2(v)v^2 - 2\gamma^2(v)\boldsymbol{v}\cdot\boldsymbol{u} + \gamma^2(v)\left(\frac{\boldsymbol{v}\cdot\boldsymbol{u}}{c}\right)^2\right]^{1/2}. \tag{15}$$

Thus, $u' = \alpha/\omega$ and the flux density of energy is

$$S' = \frac{\alpha\omega}{u}S, \qquad S \equiv nuE_1, \tag{16}$$

according to equations (6), (11) and (13).

Finally, using equations (10), (12), (13), (14) and (16), we obtain

$$p^0_{\text{in}} = \frac{1}{c}\sigma'_{\text{tot}} S \frac{\alpha\omega}{u}\frac{1}{\omega},$$
$$\boldsymbol{p}_{\text{in}} = \frac{1}{c}\sigma'_{\text{tot}} S \frac{\alpha\omega}{u}\frac{1}{\omega}\frac{\boldsymbol{u}}{c}. \tag{17}$$

The incident four-momentum of solar wind per unit time is

$$p^{\mu}_{\text{in}} = \frac{1}{c}\sigma'_{\text{tot}} S \frac{\alpha\omega}{u}\xi^{\mu}, \qquad \xi^{\mu} \equiv \left(\frac{1}{\omega}; \frac{1}{\omega}\frac{\boldsymbol{u}}{c}\right). \tag{18}$$

2.2 Reaction of the dust particle to the incident solar wind

The incident solar wind corpuscule can be reflected from the surface of the IDP, or it can cause the erosion/destruction of the IDP and decrease its mass, or, in general, similar processes such as reflection, absorption and diffraction can occur. The particle's loss of energy (per unit time) in the proper reference frame of the IDP can be written as $E'_{\text{out}} - E'_{\text{in}}$. Thus, E'_{out} can be written as an x'-part of the incident energy per unit time. The relation

$$E'_{\text{out}} = x'E'_{\text{in}} \tag{19}$$

holds for the outgoing energy. In order to express the outgoing momentum, we declare the orthonormal vector basis $\{\boldsymbol{f'}_j; j = 1, 2, 3\}$ in the proper reference frame of the particle and three velocity vectors $\{\boldsymbol{u'}_j = u'\boldsymbol{f'}_j; j = 1, 2, 3\}$ corresponding to these unit vectors. We suppose that $\boldsymbol{u'}_1 \equiv \boldsymbol{u'}$. Now, the outgoing momentum per unit time is

$$\boldsymbol{p'}_{\text{out}} = \boldsymbol{p'}_{\text{in}} - \sigma'_{\text{tot}}\frac{S'}{c}\sum_{j=1}^{3}\frac{\sigma'_{\text{pr},j}}{\sigma'_{\text{tot}}}\frac{\boldsymbol{u'}_j}{c}$$
$$= \left(1 - \frac{\sigma'_{\text{pr}}}{\sigma'_{\text{tot}}}\right)\boldsymbol{p'}_{\text{in}} - \sigma'_{\text{tot}}\frac{S'}{c}\sum_{j=2}^{3}\frac{\sigma'_{\text{pr},j}}{\sigma'_{\text{tot}}}\frac{\boldsymbol{u'}_j}{c}, \tag{20}$$

where $\sigma'_{\text{pr},j}$ ($j = 1, 2, 3$; $\sigma'_{\text{pr},1} \equiv \sigma'_{\text{pr}}$) are pressure cross-sections (there is an analogy with optics; see Klačka 2008a,b).

The outgoing four-momentum per unit time is given by equations (7), (19) and (20):

$$p'^{\mu}_{\text{out}} = \left(\frac{E'_{\text{out}}}{c}; \boldsymbol{p'}_{\text{out}}\right), \tag{21}$$

$$p'^{\mu}_{\text{out}} = \left[\frac{1}{c}\sigma'_{\text{tot}}S'x'; \left(1 - \frac{\sigma'_{\text{pr}}}{\sigma'_{\text{tot}}}\right)\boldsymbol{p'}_{\text{in}} - \sigma'_{\text{tot}}\frac{S'}{c}\sum_{j=2}^{3}\frac{\sigma'_{\text{pr},j}}{\sigma'_{\text{tot}}}\frac{\boldsymbol{u'}_j}{c}\right].$$

For the outgoing four-momentum per unit time in the stationary reference frame, using equation (16), the generalized special Lorentz transformation of p'^{μ}_{out} gives

$$p^{\mu}_{\text{out}} = \frac{1}{c}\sigma'_{\text{tot}} S\frac{\alpha\omega}{u}x'\frac{U^{\mu}}{c}$$
$$+ \left(1 - \frac{\sigma'_{\text{pr}}}{\sigma'_{\text{tot}}}\right)\frac{1}{c}\sigma'_{\text{tot}} S\frac{\alpha\omega}{u}\left(\xi^{\mu} - \frac{U^{\mu}}{c}\right)$$
$$- \sigma'_{\text{tot}}\frac{S}{c}\frac{\alpha\omega}{u}\sum_{j=2}^{3}\frac{\sigma'_{\text{pr},j}}{\sigma'_{\text{tot}}}\left(\xi^{\mu}_j - \frac{U^{\mu}}{c}\right), \tag{22}$$

where

$$U^{\mu} = [\gamma(v)c; \gamma(v)\boldsymbol{v}] \tag{23}$$

is four-velocity of the IDP. The other four-vectors are

$$\xi_j^{\mu} = \left(\frac{1}{\omega_j}; \frac{1}{\omega_j}\frac{\boldsymbol{u}_j}{c}\right),$$
$$\omega_j \equiv \gamma(v)\left(1 - \frac{\boldsymbol{v}\cdot\boldsymbol{u}_j}{c^2}\right),$$
$$\boldsymbol{u}_j = \left[\gamma(v)\left(1 + \frac{\boldsymbol{v}\cdot\boldsymbol{u}'_j}{c^2}\right)\right]^{-1} \times \left[\boldsymbol{u}'_j + \left\{[\gamma(v) - 1]\frac{\boldsymbol{v}\cdot\boldsymbol{u}'_j}{v^2} + \gamma(v)\right\}\boldsymbol{v}\right],$$
$$j = 1, 2, 3, \tag{24}$$

and $\omega_1 \equiv \omega$, $\xi_1^{\mu} \equiv \xi^{\mu}$, and $\boldsymbol{u}_1 \equiv \boldsymbol{u}$.

2.3 Equation of motion

Now we can write the equation of motion of the IDP under the action of the solar wind in a relativistically covariant form:

$$\frac{\mathrm{d}p^{\mu}}{\mathrm{d}\tau} = p^{\mu}_{\mathrm{in}} - p^{\mu}_{\mathrm{out}}. \tag{25}$$

Using equations (18) and (22), equation (25) yields

$$\frac{\mathrm{d}p^{\mu}}{\mathrm{d}\tau} = \frac{1}{c}\sigma'_{\mathrm{tot}}S\frac{\alpha\omega}{u} \times \left\{\frac{\sigma'_{\mathrm{pr}}}{\sigma'_{\mathrm{tot}}}\xi^{\mu} - \left[x' - \left(1 - \frac{\sigma'_{\mathrm{pr}}}{\sigma'_{\mathrm{tot}}}\right)\right]\frac{U^{\mu}}{c}\right\} + \sigma'_{\mathrm{tot}}\frac{S}{c}\frac{\alpha\omega}{u}\sum_{j=2}^{3}\frac{\sigma'_{\mathrm{pr},j}}{\sigma'_{\mathrm{tot}}}\left(\xi_j^{\mu} - \frac{U^{\mu}}{c}\right), \tag{26}$$

where $p^{\mu} = mU^{\mu}$ is the four-momentum of the IDP of mass m and τ is the proper time of the particle.

Using

$$\frac{\mathrm{d}p^{\mu}}{\mathrm{d}\tau} = \frac{\mathrm{d}}{\mathrm{d}\tau}(mU^{\mu}) = \frac{\mathrm{d}m}{\mathrm{d}\tau}U^{\mu} + m\frac{\mathrm{d}U^{\mu}}{\mathrm{d}\tau}, \tag{27}$$

equation (26) yields not only the acceleration of the particle, but also the change of the particle's (rest) mass, as a result of the interaction of the IDP with the solar wind. On the basis of equations (12), (18), (23), (26) and (27), the change of the mass is given by the expression ($U_{\mu}U^{\mu} = c^2$, $U_{\mu}\,\mathrm{d}U^{\mu}/\mathrm{d}\tau = 0$)

$$\frac{\mathrm{d}m}{\mathrm{d}\tau} = -\frac{1}{c^2}\sigma'_{\mathrm{tot}}S\frac{\alpha\omega}{u}\left(x' - 1\right). \tag{28}$$

We can easily verify that equation (28) corresponds to the famous Einstein equation $\mathrm{d}m/\mathrm{d}\tau = (E'_{\mathrm{in}} - E'_{\mathrm{out}})/c^2$, if equations (4) and (19) are also used.

Equations (26)–(28) yield for the four-acceleration of the IDP

$$\frac{\mathrm{d}U^{\mu}}{\mathrm{d}\tau} = \sigma'_{\mathrm{pr}}\frac{S}{mc}\frac{\alpha\omega}{u}\left(\xi^{\mu} - \frac{U^{\mu}}{c}\right) + \sigma'_{\mathrm{tot}}\frac{S}{mc}\frac{\alpha\omega}{u}\sum_{j=2}^{3}\frac{\sigma'_{\mathrm{pr},j}}{\sigma'_{\mathrm{tot}}}\left(\xi_j^{\mu} - \frac{U^{\mu}}{c}\right). \tag{29}$$

In the further treatment, we consider the case $\sigma'_{\mathrm{pr},j} \equiv 0$ for $j = 2$, 3. This corresponds to a dust particle with a spherically distributed mass. As a consequence, the equation of motion will be of the form

$$\frac{\mathrm{d}p^{\mu}}{\mathrm{d}\tau} = \frac{1}{c}\sigma'_{\mathrm{tot}}S\frac{\alpha\omega}{u}\left\{\frac{\sigma'_{\mathrm{pr}}}{\sigma'_{\mathrm{tot}}}\xi^{\mu} - \left[x' - \left(1 - \frac{\sigma'_{\mathrm{pr}}}{\sigma'_{\mathrm{tot}}}\right)\right]\frac{U^{\mu}}{c}\right\}, \tag{30}$$

and the four-acceleration will be

$$\frac{\mathrm{d}U^{\mu}}{\mathrm{d}\tau} = \frac{\sigma'_{\mathrm{pr}}S}{mc}\frac{\alpha\omega}{u}\left(\xi^{\mu} - \frac{U^{\mu}}{c}\right). \tag{31}$$

The total scattering cross-section for the spherical particle is given in Appendix A. The change of the particle's mass is given by equation (28). The conventional approach is that the force, resulting from the solar wind bombardment, considers only the fixed mass of the particle (e.g. Mukai & Yamamoto 1982).

2.4 Equation of motion to the second order in v/u

In the approximation to the first order in v/c, we can replace the space-like part of the four-acceleration of the IDP by acceleration $\mathrm{d}\boldsymbol{v}/\mathrm{d}t$, where t is the time measured in the stationary reference frame (associated with the Sun). Furthermore, using equations (3), (12), (15), (16), (18) and (23), we can express the right-hand side of equation (31) in the approximation to the second order in v/u. We obtain

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{A'nm_1u^2}{m}\left[\left(1 - \frac{\boldsymbol{v}\cdot\hat{\boldsymbol{u}}}{u}\right)\hat{\boldsymbol{u}} - \frac{\boldsymbol{v}}{u} + \frac{1}{2}\frac{v^2}{u^2}\hat{\boldsymbol{u}} + \frac{\boldsymbol{v}\cdot\hat{\boldsymbol{u}}}{u}\frac{\boldsymbol{v}}{u} - \frac{1}{2}\frac{(\boldsymbol{v}\cdot\hat{\boldsymbol{u}})^2}{u^2}\hat{\boldsymbol{u}}\right],$$
$$A' = \pi R^2 = \sigma'_{\mathrm{pr}}, \tag{32}$$

where $\hat{\boldsymbol{u}} \equiv \boldsymbol{u}/u$ is the unit vector in the direction of the solar wind.

Let us introduce the orthogonal coordinate system associated with the orbital plane of the IDP and determined by unit vectors $\boldsymbol{e}_{\mathrm{R}}$ (radial vector), $\boldsymbol{e}_{\mathrm{T}}$ (transversal vector) and $\boldsymbol{e}_{\mathrm{N}} = \boldsymbol{e}_{\mathrm{R}}\times\boldsymbol{e}_{\mathrm{T}}$ (normal vector). We can write (Klačka 1994)

$$\hat{\boldsymbol{u}} = \gamma_{\mathrm{R}}\boldsymbol{e}_{\mathrm{R}} + \gamma_{\mathrm{T}}\hat{\boldsymbol{u}}_{\mathrm{T}}, \tag{33}$$

where

$$\gamma_{\mathrm{R}} = \cos\varepsilon, \qquad \gamma_{\mathrm{T}} = \sin\varepsilon \tag{34}$$

and

$$\hat{\boldsymbol{u}}_{\mathrm{T}} = \frac{1}{N}\boldsymbol{k}\times\boldsymbol{e}_{\mathrm{R}} = \frac{1}{N}\left(\boldsymbol{e}_{\mathrm{T}}\cos i - \boldsymbol{e}_{\mathrm{N}}\cos\Theta\sin i\right),$$
$$N = \sqrt{(\cos i)^2 + (\cos\Theta)^2(\sin i)^2}. \tag{35}$$

The quantity ε is an angle between the radial direction and the real direction of the solar wind. The unit vector $\boldsymbol{k}$ corresponds to the solar rotation angular velocity vector. The inclination of the orbital plane of the IDP with respect to the solar equatorial plane is i. Finally, Θ is a position angle of the IDP (an angle measured from the ascending node of the orbit of the IDP to its actual position).

Inserting equations (33) and (35) into equation (32), and using the decomposition of the velocity vector into its radial and transversal components, $\boldsymbol{v} = v_{\mathrm{R}}\boldsymbol{e}_{\mathrm{R}} + v_{\mathrm{T}}\boldsymbol{e}_{\mathrm{T}}$, we obtain

$$\begin{aligned}
\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} &= \frac{A' n m_1 u^2}{m} \left[\left(X\gamma_{\mathrm{R}} - Y\frac{v_{\mathrm{R}}}{u} \right) \boldsymbol{e}_{\mathrm{R}} \right. \\
&\quad + \left(X\gamma_{\mathrm{T}} \frac{\cos i}{N} - Y\frac{v_{\mathrm{T}}}{u} \right) \boldsymbol{e}_{\mathrm{T}} \\
&\quad \left. - X\gamma_{\mathrm{T}} \frac{\cos\Theta \sin i}{N} \boldsymbol{e}_{\mathrm{N}} \right], \\
X &= 1 - \gamma_{\mathrm{R}}\frac{v_{\mathrm{R}}}{u} - \gamma_{\mathrm{T}}\frac{\cos i}{N}\frac{v_{\mathrm{T}}}{u} + \frac{1}{2}\frac{v^2}{u^2} \\
&\quad - \frac{1}{2}\left(\gamma_{\mathrm{R}}\frac{v_{\mathrm{R}}}{u} + \gamma_{\mathrm{T}}\frac{\cos i}{N}\frac{v_{\mathrm{T}}}{u} \right)^2, \\
Y &= 1 - \gamma_{\mathrm{R}}\frac{v_{\mathrm{R}}}{u} - \gamma_{\mathrm{T}}\frac{\cos i}{N}\frac{v_{\mathrm{T}}}{u}. \qquad (36)
\end{aligned}$$

The angle ε is small; its value lies between 2° and 3° (Bruno et al. 2003). Thus, we can neglect terms proportional to γ_{T}^2 and $\gamma_{\mathrm{T}}(v/u)^2$. Similarly, we put $\gamma_{\mathrm{R}} \approx 1$. Then

$$\begin{aligned}
\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} &= \frac{A' n m_1 u^2}{m} \left\{ \left(1 - 2\frac{v_{\mathrm{R}}}{u} \right. \right. \\
&\quad \left. - \gamma_{\mathrm{T}}\frac{\cos i}{N}\frac{v_{\mathrm{T}}}{u} + \frac{1}{2}\frac{v_{\mathrm{T}}^2}{u^2} + \frac{v_{\mathrm{R}}^2}{u^2} \right) \boldsymbol{e}_{\mathrm{R}} \\
&\quad + \left[\left(1 - \frac{v_{\mathrm{R}}}{u} \right) \gamma_{\mathrm{T}}\frac{\cos i}{N} - \frac{v_{\mathrm{T}}}{u} + \frac{v_{\mathrm{R}} v_{\mathrm{T}}}{u^2} \right] \boldsymbol{e}_{\mathrm{T}} \\
&\quad \left. - \left(1 - \frac{v_{\mathrm{R}}}{u} \right) \gamma_{\mathrm{T}} \frac{\cos\Theta \sin i}{N} \boldsymbol{e}_{\mathrm{N}} \right\}. \qquad (37)
\end{aligned}$$

3 EQUATION OF MOTION: ELECTROMAGNETIC RADIATION EFFECT

Until now, we have dealt with the action of the solar wind on the motion of an IDP. The role of solar electromagnetic radiation cannot be neglected in the motion of the IDP in the Solar system. The relativistically covariant equation of motion for an arbitrarily shaped IDP under the action of a parallel beam of photons is (Klačka 2008a,b)

$$\frac{\mathrm{d}p^{\mu}}{\mathrm{d}\tau} = \sum_{j=1}^{3} \left(\frac{w_1^2 S_{\mathrm{elmg}} \bar{C}'_{\mathrm{pr},j}}{c^2} + \frac{1}{c} F'_{e,j} \right) \left(c b_j^{\mu} - U^{\mu} \right). \qquad (38)$$

Here, p^{μ} is the four-momentum of the particle of mass m, the four-vector of the world-velocity of the particle is given by equation (23) and the four-vectors $b_j^{\mu}, j = 1, 2, 3$ are given as

$$\begin{aligned}
b_j^{\mu} &= \left(\frac{1}{w_j}; \frac{\boldsymbol{e}_j}{w_j} \right), \\
w_j &= \gamma(v) \left(1 - \frac{\boldsymbol{v}\cdot\boldsymbol{e}_j}{c} \right), \\
\boldsymbol{e}_j &= \left[\gamma(v) \left(1 + \frac{\boldsymbol{v}\cdot\boldsymbol{e'}_j}{c} \right) \right]^{-1} \\
&\quad \times \left[\boldsymbol{e'}_j + \left\{ [\gamma(v) - 1] \frac{\boldsymbol{v}\cdot\boldsymbol{e'}_j}{v^2} + \frac{\gamma(v)}{c} \right\} \boldsymbol{v} \right], \\
j &= 1, 2, 3. \qquad (39)
\end{aligned}$$

Here, $\{\boldsymbol{e'}_j; j = 1, 2, 3\}$ is the orthonormal vector basis in the proper reference frame of the particle and $\{\boldsymbol{e}_j; j = 1, 2, 3\}$ is the corresponding vector basis in the stationary frame; $\boldsymbol{e}_1$ corresponds to the radial direction (i.e. the Sun–particle direction). S_{elmg} is the flux density of the electromagnetic radiation and $\bar{C}'_{\mathrm{pr},j}$ $(j = 1, 2, 3)$ are the spectrally averaged cross-sections of radiation pressure

$$\bar{C}'_{\mathrm{pr},j} = \frac{\int_0^{\infty} I(\lambda)\, C'_{\mathrm{pr},j}(\lambda)\, \mathrm{d}\lambda}{\int_0^{\infty} I(\lambda)\, \mathrm{d}\lambda}, \qquad j = 1, 2, 3, \qquad (40)$$

where $I(\lambda)$ is the flux of monochromatic radiation energy. If $\bar{C}'_{\mathrm{pr},2} = \bar{C}'_{\mathrm{pr},3} \equiv 0$, then equation (38) reduces to the P–R effect (Poynting 1903; Robertson 1937; Klačka 2008a,b; Klačka et al. 2009). This is because, in this case, the components of the thermal emission force $F'_{e,j}$ $(j = 1, 2, 3)$ are also equal to zero (Mishchenko 2001; Mishchenko, Travis & Lacis 2002). We can easily verify that equation (38) yields $\mathrm{d}m/\mathrm{d}\tau = 0$ (i.e. that the mass of the particle is conserved, under the action of electromagnetic radiation).

To the first order in $\boldsymbol{v}/c$, equation (38) yields

$$\begin{aligned}
\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} &= \frac{S_{\mathrm{elmg}}}{mc} \sum_{j=1}^{3} \bar{C}'_{\mathrm{pr},j} \left[\left(1 - 2\frac{\boldsymbol{v}\cdot\boldsymbol{e}_1}{c} + \frac{\boldsymbol{v}\cdot\boldsymbol{e}_j}{c} \right) \boldsymbol{e}_j - \frac{\boldsymbol{v}}{c} \right] \\
&\quad + \frac{1}{m} \sum_{j=1}^{3} F'_{e,j} \left[\left(1 + \frac{\boldsymbol{v}\cdot\boldsymbol{e}_j}{c} \right) \boldsymbol{e}_j - \frac{\boldsymbol{v}}{c} \right], \\
\boldsymbol{e}_j &= \left(1 - \frac{\boldsymbol{v}\cdot\boldsymbol{e'}_j}{c} \right) \boldsymbol{e'}_j + \frac{\boldsymbol{v}}{c}, \qquad j = 1, 2, 3. \qquad (41)
\end{aligned}$$

It is worth emphasizing that the values of the radiation pressure cross-sections $\bar{C}'_{\mathrm{pr},j}, j = 1, 2, 3$, depend on the particle's orientation with respect to the incident radiation (i.e. their values are time-dependent, in general). The general equation of motion, represented by equation (38) or equation (41), differs from the P–R effect. Equations (38)–(41) hold for arbitrarily shaped particles. Krauss & Wurm (2004) have presented experimental evidence that non-spherical dust grains move in a different way to spherical particles.

As is usual in studies of the Solar system, we restrict ourselves to the P–R effect, as for the effect of solar electromagnetic radiation. Thus, instead of equation (41), we use

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{S_{\mathrm{elmg}} A' \bar{Q}'_{\mathrm{pr},1}}{mc} \left[\left(1 - \frac{\boldsymbol{v}\cdot\boldsymbol{e}_1}{c} \right) \boldsymbol{e}_1 - \frac{\boldsymbol{v}}{c} \right], \qquad (42)$$

where a dimensionless efficiency factor of radiation pressure $\bar{Q}'_{\mathrm{pr},1}$ is defined by the relation $\bar{Q}'_{\mathrm{pr},1} = \bar{C}'_{\mathrm{pr},1}/A'$. The values of $\bar{Q}'_{\mathrm{pr},1}$ can be calculated according to Mie (1908); see also van de Hulst (1981) and Bohren & Huffman (1983).

3.1 Electromagnetic radiation effect as a special case of the solar wind effect

The transformations (i) $\boldsymbol{u} \to c\boldsymbol{e}_1$, (ii) $S' = S\alpha\omega/u \to S'_{\mathrm{elmg}} = S_{\mathrm{elmg}} w_1^2$ (Klačka 2008b), (iii) $\sigma'_{\mathrm{pr},j} \to \bar{C}'_{\mathrm{pr},j}$ $(j = 1, 2, 3)$ and (iv) $x' = 1$ (Klačka 2008b) reduce equation (26) into equation (38) without thermal emission terms ($F'_{e,j} \equiv 0$ for $j = 1, 2, 3$). This means that our theory for the electromagnetic radiation effect without thermal emission is consistent with the theory for the corpuscular radiation effect.

4 EQUATION OF MOTION: SOLAR RADIATION AND SOLAR GRAVITY

Let us consider a spherical body orbiting the Sun under the action of solar radiation, i.e. solar corpuscular (solar wind) and electromagnetic radiation. The effect of the solar electromagnetic radiation on the motion of a spherical particle corresponds to the P–R effect.

4.1 Solar electromagnetic radiation effect

It is useful to introduce a β-parameter, defined as the ratio of the radial component of the radiation force and the gravitational force between the Sun and the particle with zero velocity:

$$\beta = \frac{L_\odot A' \bar{Q}'_{pr}}{4\pi c m \mu}, \qquad \mu \equiv G \left(M_\odot + m \right) \doteq G M_\odot. \tag{43}$$

$L_\odot = 3.842 \times 10^{26}$ W (Bahcall 2002) is the luminosity of the Sun, $\bar{Q}'_{pr} \equiv \bar{Q}'_{pr,1}$, G is the gravitational constant and $M_\odot$ is the mass of the Sun. For a homogeneous spherical particle, we can write

$$\beta = 5.760 \times 10^2 \frac{\bar{Q}'_{pr}}{R(\mu m) \varrho (kg\, m^{-3})}, \tag{44}$$

where R is the radius of the particle and ϱ is the mass density of the particle. Conventionally, it is assumed that $\beta =$ const: neither the optical properties nor the mass of the IDP change. We do not restrict ourselves to the validity of this assumption.

Now, on the basis of the decomposition of the velocity vector $\boldsymbol{v} = v_R \boldsymbol{e}_R + v_T \boldsymbol{e}_T$ and equation (43), we can rewrite equation (42) as

$$\left(\frac{d\boldsymbol{v}}{dt} \right)_{P-R} = \beta \frac{\mu}{r^2} \left[\left(1 - 2 \frac{v_R}{c} \right) \boldsymbol{e}_R - \frac{v_T}{c} \boldsymbol{e}_T \right]. \tag{45}$$

Here, r is the heliocentric distance of the IDP and the relation $S_{elmg} = L_\odot/(4\pi r^2)$ is used. This is the acceleration of the IDP under the action of the P–R effect.

4.2 Solar wind effect

Let us replace the fraction before the curly braces in equation (37) by the following new quantities:

$$\frac{A' n m_1 u^2}{m} \equiv \frac{\eta}{\bar{Q}'_{pr}} \beta \frac{u}{c} \frac{\mu}{r^2}. \tag{46}$$

Here, η is conventionally a constant with a value of 0.22 (Whipple 1967; Dohnanyi 1978) or 0.30 (Gustafson 1994; Abe 2009). The solar wind speed u values of $350\,km\,s^{-1}$ (Dohnanyi 1978) or $400\,km\,s^{-1}$ (Gustafson 1994) are used to model the orbital evolution under the action of solar wind.

Let us look at the numerical values of η and u on the basis of the solar physics data. In order to calculate the value of η, we need the values of n, u and E_1, on the basis of equation (16). The average values near the orbit of the Earth (1 au) are (Hundhausen 1997, p. 92): proton density $n_1 = 6.6$ cm^{-3}; electron density $n_2 = 7.1$ cm^{-3}; He^{2+} density $n_3 = 0.25$ cm^{-3}; flow speed $u = 450\,km\,s^{-1}$. For the average value of S for the solar wind (subscript 'sw') at 1 au, equation (16) yields

$$S_{sw} = u \sum_{i=1}^{3} n_i E_{1i} \doteq c^2 u \sum_{i=1}^{3} n_i m_i = 515.642\,kg\,s^{-3}.$$

Moreover, we take into account that $n_i u$ (1 au) = $\langle n_i \rangle \langle u \rangle$ (1 au) $(1 - 0.15 \cos\varphi)^2$, $\varphi = 2\pi[t - t_{max}]/T$, where $T = 11.1$ yr and t_{max} is the time of the solar cycle maximum (Svalgaard 1977, chap. 13). This result, together with equations (3) ($\gamma\,(u) \doteq 1$), (16), (32), (43) and (46) yields $S_{sw} = S_{elmg}\,(A'/\sigma'_{pr})\eta$ and

$$\eta = \eta_0 \left(1 - \delta \cos\varphi\right)^2, \qquad u = u_0 \left(1 - \delta \cos\varphi\right),$$
$$\eta_0 = 0.38, \qquad \delta = 0.15, \qquad u_0 = 450\,km\,s^{-1},$$
$$\varphi = 2\pi \frac{t - t_{retard} - t_{max}}{T}, \qquad T = 11.1\,yr, \tag{47}$$

if we put $\sigma_{pr} = A'$ and S_{elmg} (1 au) $= L_\odot/[4\pi(1\,au)^2]$, $L_\odot = 3.842 \times 10^{26}$ W. The value of T represents the average value of the solar cycle period (e.g. Foukal 2004, p. 366). The retarded time t_{retard} is of the order of r/u_0, which is only a better approximation to reality than the omission of this term.

4.2.1 More exact solution

In reality, we need to know the concentration $n(r, t)$ and solar wind velocity $u(r, t)$, when the dust grain is situated at the position of heliocentric distance r at time t. More precise information can be obtained from the continuity equation (the radial component of the velocity $\boldsymbol{u}$ is approximated by the magnitude u)

$$\frac{\partial n}{\partial t} + \frac{1}{r^2} \frac{\partial \left(r^2 n u \right)}{\partial r} = 0, \tag{48}$$

if $r \neq 0$. Using the observational fact $n =$ const u/r^2 (Svalgaard 1977, chap. 13), we obtain

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial r} = 0, \tag{49}$$

if $r \neq 0$. Using the boundary condition

$$\lim_{r \to 0} u(r, t) = u_0 \left[1 - \delta \cos \left(2\pi \frac{t - t_{max}}{T} \right) \right], \tag{50}$$

the quasi-linear partial differential equation (49) can be solved.

Equation (49) is known as the Burgers equation:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0,$$

(e.g. Ševčovič 2008, pp. 40–42). If the boundary condition

$$u(x = 0, t) = \chi(t)$$

is given, then the solution of the Burgers equation is

$$u(x, t) = \chi \left[t - \frac{x}{u(x, t)} \right].$$

The last non-linear algebraic equation can be solved by an iteration method, e.g.

$$u = \chi \left(t - \frac{x}{u_0} \right) + \lim_{k \to \infty} v_k,$$

$$v_{k+1} = \chi \left[t - \frac{x}{\chi(t - x/u_0) + v_k} \right] - \chi \left(t - \frac{x}{u_0} \right),$$

$$v_1 = 0.$$

On the basis of the known solution of the Burgers equation, equations (49) and (50) yield

$$u(r, t) = u_0 \left[1 - \delta \cos \left\{ 2\pi \frac{t - r/[2u(r, t)] - t_{max}}{T} \right\} \right]. \tag{51}$$

A comparison with equation (47) shows that the retarded time is $t_{retard} = r/(2u)$.

The non-linear algebraic equation (51) can be solved using the iteration method presented above, or by using the following iteration

$$u_{k+1}(r, t) = u_0 \left[1 - \delta \cos \left\{ 2\pi \frac{t - r/[2u_k(r, t)] - t_{max}}{T} \right\} \right], \tag{52}$$

because the right-hand side of equation (51) is a contractive/contraction function for the case $u^2 > \pi u_0 r \delta/T$ and the heliosphere is characterized by the condition $r < 150$ au (approximately). We can put $u_1(r, t) = u_0$.

4.2.2 Summary

Using the definition of equation (46), on the basis of equations (47) and (51), we can summarize

$$\frac{A' n m_1 u^2}{m} \equiv \frac{\eta}{\bar{Q}'_{\rm pr}} \beta \frac{u}{c} \frac{\mu}{r^2},$$

$$u(r,t) = u_0 \left[1 - \delta \cos \left\{ 2\pi \frac{t - r/[2u(r,t)] - t_{\max}}{T} \right\} \right],$$

$$\eta(r,t) = \eta_0 \left[\frac{u(r,t)}{u_0} \right]^2, \qquad \eta_0 = 0.38,$$

$$\delta = 0.15, \qquad u_0 = 450\,{\rm km\,s^{-1}}, \qquad T = 11.1\,{\rm yr}. \tag{53}$$

We can also use

$$n \equiv n(r,t) = n_0 \left[\frac{u(r,t)}{u_0} \right] \left[\frac{1\,{\rm au}}{r({\rm au})} \right]^2 .$$

These results represent a more realistic model than the model that is conventionally used, because it takes into account more observational facts (Svalgaard 1977; Hundhausen 1997).

For more simple cases, equation (47) is used with $t_{\rm retard} = r/(2u_0)$ or $t_{\rm retard} = 0$. Shock waves do not exist in these simple cases. The shock waves are generated by the solution of the Burgers equation, which is considered in equation (53). As pointed out in the comment to equation (52), the shock waves are not realized in the Solar system.

We put $i = 0$ in equation (37). Then, the acceleration of the IDP caused by the solar wind has the form

$$\begin{aligned} \left(\frac{{\rm d}\boldsymbol{v}}{{\rm d}t} \right)_{\rm SW} = \frac{\eta}{\bar{Q}'_{\rm pr}} \beta \frac{\mu}{r^2} \Bigg\{ & \left(\frac{u}{c} - 2\frac{v_{\rm R}}{c} \right) \boldsymbol{e}_{\rm R} - \frac{v_{\rm T}}{c} \boldsymbol{e}_{\rm T} \\ & - \gamma_{\rm T} \left[\frac{v_{\rm T}}{c} \boldsymbol{e}_{\rm R} - \left(\frac{u}{c} - \frac{v_{\rm R}}{c} \right) \boldsymbol{e}_{\rm T} \right] \\ & + \frac{1}{2} \frac{v_{\rm T}^2}{uc} \boldsymbol{e}_{\rm R} + \frac{v_{\rm R}}{c} \frac{\boldsymbol{v}}{u} \Bigg\}. \end{aligned} \tag{54}$$

4.3 Equation of motion

The gravitational acceleration from the Sun is $-(\mu/r^2)\boldsymbol{e}_{\rm R}$. In order to obtain the final equation of motion of the IDP, we sum the gravitational acceleration from the Sun, equations (45) and (54) and we obtain

$$\begin{aligned} \frac{{\rm d}\boldsymbol{v}}{{\rm d}t} = & -\frac{\mu}{r^2} \left(1 - \beta - \frac{\eta}{\bar{Q}'_{\rm pr}} \beta \frac{u}{c} \right) \boldsymbol{e}_{\rm R} - \left(1 + \frac{\eta}{\bar{Q}'_{\rm pr}} \right) \beta \frac{\mu}{r^2} \\ & \times \left(2\frac{v_{\rm R}}{c} \boldsymbol{e}_{\rm R} + \frac{v_{\rm T}}{c} \boldsymbol{e}_{\rm T} \right) + \frac{\eta}{\bar{Q}'_{\rm pr}} \beta \frac{\mu}{r^2} \Bigg\{ -\gamma_{\rm T} \frac{v_{\rm T}}{c} \boldsymbol{e}_{\rm R} \\ & + \gamma_{\rm T} \left(\frac{u}{c} - \frac{v_{\rm R}}{c} \right) \boldsymbol{e}_{\rm T} + \frac{1}{2} \frac{v_{\rm T}^2}{uc} \boldsymbol{e}_{\rm R} + \frac{v_{\rm R}}{c} \frac{\boldsymbol{v}}{u} \Bigg\}. \end{aligned} \tag{55}$$

For the constant solar wind and the dust particle for which $(\eta/\bar{Q}'_{\rm pr})(u/c) \ll 1$, we can neglect the solar wind pressure term $(\eta/\bar{Q}'_{\rm pr})\beta(u/c)(\mu/r^2)\,\boldsymbol{e}_{\rm R}$ with respect to the solar electromagnetic radiation pressure term $\beta(\mu/r^2)\,\boldsymbol{e}_{\rm R}$ in equation (55). However, for the (time-)variable solar wind (equation 47), the variable term $(\eta/\bar{Q}'_{\rm pr})\beta(u/c)(\mu/r^2)\,\boldsymbol{e}_{\rm R}$ can be dominant with respect to other variable terms caused by the solar wind. Therefore, the term cannot be neglected. Hence, we use equation (55) for the variable solar wind and we use equation (55), with the neglected solar wind pressure term, for the constant solar wind. For both the constant and the variable solar wind, we can neglect the solar wind pressure term with respect to the solar electromagnetic radiation pressure term when calculating the orbital elements, if $(\eta/\bar{Q}'_{\rm pr})(u/c) \ll 1$. Therefore, we introduce a new central acceleration $-\mu(1-\beta)\,\boldsymbol{e}_{\rm R}/r^2$ for the calculation of orbital elements (i.e. the gravitational acceleration from the Sun reduced by the solar electromagnetic radiation pressure).

From equation (28) we can easily see that the mass-loss rate per unit surface for a spherical dust grain is independent of the particle radius. This is in accordance with the results of Whipple (1955), Dohnanyi (1978) and Mukai & Schwehm (1981). Moreover, equation (28) can be rewritten in a form describing the decrease of the particle radius R:

$$\frac{{\rm d}R}{{\rm d}t} = -\frac{K}{r^2} \frac{|\boldsymbol{u} - \boldsymbol{v}|}{u} (1 - \delta \cos\varphi)^2 ,$$

$$u = u_0 (1 - \delta \cos\varphi), \qquad \delta = 0.15, \qquad u_0 = 450\,{\rm km\,s^{-1}},$$

$$\varphi = 2\pi \frac{t - t_{\rm retard} - t_{\max}}{T}, \qquad T = 11.1\,{\rm yr}, \tag{56}$$

Here, K is a constant characterizing the decrease of the radius of the particle; we also used equation (47) (see also the text between equations 47 and 54). Equation (34) also has to be used: $\boldsymbol{u} = u\hat{\boldsymbol{u}}$. We can use equation (56) as an approximation to the process of erosion of the particle because of the solar wind corpuscles. Because β is a function of R ($\bar{Q}'_{\rm pr}$ is also a function of R), equations (47), (55) and (56) have to be solved simultaneously, together with the Mie calculations, yielding $\bar{Q}'_{\rm pr}$ for a given R.

The approximation of a constant flux can be described by the approximations

$$\eta \doteq \eta_0 = 0.38, \qquad u \doteq u_0 = 450\,{\rm km\,s^{-1}}. \tag{57}$$

If we neglect the terms of the second order in v/u and the solar wind pressure term in equation (55), the equation of motion of the dust particle, under the action of constant radial solar wind ($\gamma_{\rm T} \equiv 0$) and the P–R effect in the Sun's gravitational field equation of motion, is

$$\begin{aligned} \frac{{\rm d}\boldsymbol{v}}{{\rm d}t} = & -\frac{\mu}{r^2} (1 - \beta)\, \boldsymbol{e}_{\rm R} \\ & - \left(1 + \frac{\eta_0}{\bar{Q}'_{\rm pr}} \right) \beta \frac{\mu}{r^2} \left(2\frac{v_{\rm R}}{c} \boldsymbol{e}_{\rm R} + \frac{v_{\rm T}}{c} \boldsymbol{e}_{\rm T} \right). \end{aligned} \tag{58}$$

An analytical approach to the solution of equations (47), (55) and (56) is presented in Section 5. Detailed numerical solutions are given in Section 6.

5 SECULAR EVOLUTION OF A PARTICLE'S ORBITAL ELEMENTS UNDER THE ACTION OF SOLAR RADIATION: ANALYTICAL APPROACH

We have obtained the complete equation of motion in Section 4. Our task in this section is to gain a qualitative understanding of the orbital evolution of an IDP.

5.1 Calculation of secular time derivatives of orbital elements

Let us use the perturbation equations of celestial mechanics in the following forms:

$$
\begin{aligned}
\frac{da_\beta}{dt} &= \frac{a_\beta}{1-e_\beta^2}\left\{2\sqrt{\frac{p_\beta}{\mu(1-\beta)}}\right.\\
&\quad\times\left[a_R e_\beta \sin f_\beta + a_T\left(1+e_\beta\cos f_\beta\right)\right]\\
&\quad\left.+\frac{\dot\beta}{1-\beta}\left(1+e_\beta^2+2e_\beta\cos f_\beta\right)\right\};\\
\frac{de_\beta}{dt} &= \sqrt{\frac{p_\beta}{\mu(1-\beta)}}\\
&\quad\times\left[a_R\sin f_\beta + a_T\left(\cos f_\beta+\frac{e_\beta+\cos f_\beta}{1+e_\beta\cos f_\beta}\right)\right]\\
&\quad+\frac{\dot\beta}{1-\beta}\left(e_\beta+\cos f_\beta\right);\\
\frac{d\omega_\beta}{dt} &= -\sqrt{\frac{p_\beta}{\mu(1-\beta)}}\frac{1}{e_\beta}\\
&\quad\times\left(a_R\cos f_\beta - a_T\sin f_\beta\frac{2+e_\beta\cos f_\beta}{1+e_\beta\cos f_\beta}\right)\\
&\quad+\frac{1}{e_\beta}\frac{\dot\beta}{1-\beta}\sin f_\beta. \qquad (59)
\end{aligned}
$$

Here, $p_\beta = a_\beta(1-e_\beta^2)$, f_β is the true anomaly of the IDP and ω_β is the argument of perihelion of the particle's orbit; the dot over β denotes differentiation with respect to time. It is assumed that the longitude of the ascending node is time-independent. The index β denotes that a quantity is calculated for central acceleration $-[\mu(1-\beta)/r^2]\boldsymbol{e}_R$. Other terms on the right-hand side of equation (55) with the neglected solar wind pressure term constitute the non-gravitational disturbing acceleration. Moreover, a_R and a_T are the radial and transversal components of the disturbing acceleration, respectively. Equations (59) are consistent with equations (12) and (14) of Klačka (1993a) and equations (32), (34) and (37) of Klačka (1993b), if $M = M_\odot(1-\beta)$. Using equation (55) and the following expressions

$$
\begin{aligned}
v_R &= \sqrt{\frac{\mu(1-\beta)}{p_\beta}}e_\beta\sin f_\beta,\\
v_T &= \sqrt{\frac{\mu(1-\beta)}{p_\beta}}\left(1+e_\beta\cos f_\beta\right), \qquad (60)
\end{aligned}
$$

we obtain

$$
\begin{aligned}
a_R &= -2\left(1+\frac{\eta}{\bar{Q}'_{pr}}\right)\beta\frac{\mu}{r^2}\frac{\sqrt{\mu(1-\beta)/p_\beta}}{c}e_\beta\sin f_\beta\\
&\quad+\frac{\eta}{\bar{Q}'_{pr}}\beta\frac{\mu}{r^2}\frac{1}{c}\left\{-\gamma_T\sqrt{\frac{\mu(1-\beta)}{p_\beta}}\left(1+e_\beta\cos f_\beta\right)\right.\\
&\quad+\frac{\mu(1-\beta)/p_\beta}{u}\\
&\quad\left.\times\left[\frac{1}{2}\left(1+2e_\beta\cos f_\beta+e_\beta^2\cos^2 f_\beta\right)+e_\beta^2\sin^2 f_\beta\right]\right\},\\
a_T &= -\left(1+\frac{\eta}{\bar{Q}'_{pr}}\right)\beta\frac{\mu}{r^2}\frac{\sqrt{\mu(1-\beta)/p_\beta}}{c}\left(1+e_\beta\cos f_\beta\right)\\
&\quad+\frac{\eta}{\bar{Q}'_{pr}}\beta\frac{\mu}{r^2}\frac{1}{c}\left[\gamma_T u-\gamma_T\sqrt{\frac{\mu(1-\beta)}{p_\beta}}e_\beta\sin f_\beta\right.\\
&\quad\left.+\frac{\mu(1-\beta)/p_\beta}{u}e_\beta\left(1+e_\beta\cos f_\beta\right)\sin f_\beta\right]. \qquad (61)
\end{aligned}
$$

We obtain the secular evolution of the orbital element g by the time averaging of dg/dt over one orbital period P, i.e.

$$
\begin{aligned}
\left\langle\frac{dg}{dt}\right\rangle &\equiv \frac{1}{P}\int_0^P\frac{dg}{dt}\,dt\\
&= \frac{1}{a_\beta^2\sqrt{1-e_\beta^2}}\frac{1}{2\pi}\int_0^{2\pi}r^2\frac{dg}{dt}\left(f_\beta\right)\,df_\beta. \qquad (62)
\end{aligned}
$$

We have used the second and third Kepler laws

$$
r^2\frac{df_\beta}{dt} = \sqrt{\mu(1-\beta)p_\beta}-\frac{d\omega_\beta}{dt}-\frac{d\Omega_\beta}{dt}\cos i_\beta \doteq \sqrt{\mu(1-\beta)p_\beta}
$$

and

$$
\frac{a_\beta^3}{P^2} = \frac{\mu(1-\beta)}{4\pi^2},
$$

where ω_β is the argument of perihelion and Ω_β is the longitude of the ascending node.

The quantity $\dot\beta$ is a function of the radius R of the particle. Using an approximation $\beta = A/R + B$, where A and B are constants for a given particle, we can write

$$
\beta = \frac{A}{R}+B, \qquad \frac{d\beta}{dt} = -\frac{A}{R^2}\frac{dR}{dt} = \frac{A}{R^2}\frac{K}{r^2}, \qquad (63)
$$

if the dominant part of equation (56) is also used ($\delta = 0$, $|\boldsymbol{u}-\boldsymbol{v}| \doteq u$).

If we apply equation (62) to equations (59) and (63), assuming that the orbital elements and the β-parameter do not significantly change during a particle's revolution around the Sun, we obtain ($\delta \equiv 0$ is assumed)

$$
\begin{aligned}
\frac{da_\beta}{dt} &= -\beta\frac{\mu}{c}\frac{2+3e_\beta^2}{a_\beta\left(1-e_\beta^2\right)^{3/2}}\\
&\quad\times\left\{1+\frac{\eta_0}{\bar{Q}'_{pr}}\left[1-2\gamma_T\frac{1}{2+3e_\beta^2}\frac{u_0}{\sqrt{\mu(1-\beta)/p_\beta}}\right]\right\}\\
&\quad+\frac{1}{1-\beta}\frac{A}{R^2}K\frac{1+e_\beta^2}{a_\beta\left(1-e_\beta^2\right)^{3/2}},\\
\frac{de_\beta}{dt} &= -\beta\frac{\mu}{c}\frac{5e_\beta/2}{a_\beta^2\sqrt{1-e_\beta^2}}\left\{1+\frac{\eta_0}{\bar{Q}'_{pr}}\right.\\
&\quad\left.\times\left[1-\frac{2}{5}\gamma_T\frac{1-\sqrt{1-e_\beta^2}}{e_\beta^2}\frac{u_0}{\sqrt{\mu(1-\beta)/p_\beta}}\right]\right\}\\
&\quad+\frac{1}{1-\beta}\frac{A}{R^2}K\frac{e_\beta}{a_\beta^2\sqrt{1-e_\beta^2}},\\
\frac{d\omega_\beta}{dt} &= -\frac{\eta_0}{\bar{Q}'_{pr}}\beta\frac{\mu}{c}\frac{1}{a_\beta^2\sqrt{1-e_\beta^2}}\\
&\quad\times\left[\gamma_T\frac{1-\sqrt{1-e_\beta^2}}{e_\beta^2}-\frac{1}{2}\frac{\sqrt{\mu(1-\beta)/p_\beta}}{u_0}\right]. \qquad (64)
\end{aligned}
$$

Here, we have omitted the symbol ⟨ ⟩ on the left-hand sides. Equations (64) hold, assuming $\eta \equiv \eta_0$.

The acceleration caused by the solar wind was used in Klačka et al. (2008) and Pástor et al. (2009a) without one term of the second order in v/u, as follows from the covariant formulation and equations (32) and (55). However, as we see, the new term does not have any influence on the secular considerations in Klačka et al. (2008) and Pástor et al. (2009a).

5.2 Complete set of differential equations for the secular evolution of a particle's orbital elements

We have obtained equations (63) and (64) for the secular evolution of orbital elements. This set of differential equations has to be completed by the equation for the secular evolution of the radius of the particle. Using equations (56) and (62), we obtain ($\delta = 0$, $|\boldsymbol{u} - \boldsymbol{v}| \doteq u$)

$$\frac{\mathrm{d}R}{\mathrm{d}t} = -\frac{K}{a_\beta^2 \sqrt{1 - e_\beta^2}}. \tag{65}$$

The complete set of differential equations for the secular evolution of the orbital elements is represented by equations (63)–(65). Initial conditions must be added to the set of differential equations. If the particle is ejected from a parent body of known orbital elements, then the particle's initial orbital elements have to be calculated from equations (60) and (61) in Klačka (2004).

If we also take into account the thermal change of the optical properties of the spherical dust particle, then further changes of the orbital elements also exist (secular changes of the semimajor axis and eccentricity, perihelion motion). This would correspond to the changes of parameters A and B in equation (63). We do not deal with this case (see Klačka et al. 2007; Pástor, Klačka & Kómar 2009b).

5.3 Discussion

Let us consider the secular evolution of the semimajor axis a_β of the particle's orbit. If the P–R effect and the radial velocity component of the solar wind are considered alone (i.e. $\gamma_\mathrm{T} \equiv 0$ and $\dot{\beta} \equiv 0$), then equations (63)–(65) show that a_β is a decreasing function of time. If we also take into account the non-radial velocity component of the solar wind, or $\dot{\beta} > 0$, the situation might be different. The secular value of a_β can be an increasing function of time. Thus, the effect of the real solar wind can cause the particle to spiral outwards from the Sun. (Similarly, the secular value of e_β can also be an increasing function of time.)

5.4 Radial solar wind and decrease of particle's radius

According to equations (28) and (56), the mass of the particle can decrease. Equation (65) holds, assuming that the decrease of the particle's radius is small enough during a particle's revolution around the Sun.

If we consider only the P–R effect and the time-independent radial solar wind effect ($\gamma_\mathrm{T} = 0$), then we are able to calculate the orbital eccentricity of the IDP as a function of the particle's radius. Using equations (64) and (65), we can immediately write

$$\frac{\mathrm{d}e_\beta}{\mathrm{d}R} = \frac{e_\beta}{K}\left[\frac{5}{2}\left(1 + \frac{\eta_0}{\bar{Q}'_\mathrm{pr}}\right)\beta\frac{\mu}{c} - \frac{1}{1-\beta}\frac{A}{R^2}K\right]. \tag{66}$$

Also using equations (44) and (63), equation (66) can be integrated as follows:

$$e_\beta = e_{\beta\mathrm{in}} \frac{A - (1-B)\,R_\mathrm{in}}{A - (1-B)\,R}\left(\frac{R}{R_\mathrm{in}}\right)^{1+k_2} \times \exp\left[k_1\left(R - R_\mathrm{in}\right)\right],$$

$$k_1 = \frac{5}{2}\frac{\mu}{c}\frac{B}{K(\mu\mathrm{m\,au}^2\,\mathrm{yr}^{-1})},$$

$$k_2 = \frac{5}{2}\frac{\mu}{c}\frac{1}{K(\mu\mathrm{m\,au}^2\,\mathrm{yr}^{-1})}\left[A + \eta \times \frac{5.760 \times 10^2}{\varrho\left(\mathrm{kg\,m}^{-3}\right)}\right],$$

$$\beta(R) = \frac{A}{R(\mu\mathrm{m})} + B,$$

$$\mu = 4\pi^2\ \mathrm{au}^3\,\mathrm{yr}^{-2}, \qquad c = 6.3114 \times 10^4\ \mathrm{au\,yr}^{-1},$$

$$\eta \equiv \eta_0 = 0.38. \tag{67}$$

The quantities A and R are given in μm and the subscript 'in' denotes initial values.

In order to find the secular evolution of the semimajor axis a_β and eccentricity e_β, we have to solve the following set of equations: equation (63) for a given values of A and B, the equation for a_β in equations (64), (65) and (67) (or the equation for e_β in equation 64 instead of equation 67).

5.5 Semilatus rectum and spiralling time for radial solar wind

From equation (64), we can also determine the secular evolution of the semilatus rectum p_β of the particle's orbit. When $p_\beta = a_\beta(1 - e_\beta^2)$, we can write

$$\left\langle\frac{\mathrm{d}p_\beta}{\mathrm{d}t}\right\rangle = \left(1 - e_\beta^2\right)\left\langle\frac{\mathrm{d}a_\beta}{\mathrm{d}t}\right\rangle - 2a_\beta e_\beta\left\langle\frac{\mathrm{d}e_\beta}{\mathrm{d}t}\right\rangle, \tag{68}$$

and using equations (64), with the symbol ⟨ ⟩ omitted, we obtain

$$\begin{aligned}\frac{\mathrm{d}p_\beta}{\mathrm{d}t} = {} & \frac{\left(1 - e_\beta^2\right)^{3/2}}{p_\beta} \\ & \times\left[-2\left(1 + \frac{\eta_0}{\bar{Q}'_\mathrm{pr}}\right)\beta\frac{\mu}{c} + \frac{1}{1-\beta}\frac{A}{R^2}K\right] \\ & + 2\gamma_\mathrm{T}\frac{\eta_0}{\bar{Q}'_\mathrm{pr}}\beta\frac{\mu}{c}\frac{u_0}{\sqrt{\mu(1-\beta)/p_\beta}}\frac{1 - e_\beta^2}{p_\beta}.\end{aligned} \tag{69}$$

Let us rewrite the equation for $\langle \mathrm{d}e_\beta/\mathrm{d}t\rangle$ in the form (with the symbol ⟨ ⟩ omitted)

$$\begin{aligned}\frac{\mathrm{d}e_\beta}{\mathrm{d}t} = {} & \frac{e_\beta\left(1 - e_\beta^2\right)^{3/2}}{p_\beta^2} \\ & \times\left[-\frac{5}{2}\left(1 + \frac{\eta_0}{\bar{Q}'_\mathrm{pr}}\right)\beta\frac{\mu}{c} + \frac{1}{1-\beta}\frac{A}{R^2}K\right] \\ & + \gamma_\mathrm{T}\frac{\eta_0}{\bar{Q}'_\mathrm{pr}}\beta\frac{\mu}{c}\frac{u_0}{\sqrt{\mu(1-\beta)/p_\beta}} \\ & \times\frac{\left(1 - \sqrt{1 - e_\beta^2}\right)\left(1 - e_\beta^2\right)^{3/2}}{p_\beta^2 e_\beta}.\end{aligned} \tag{70}$$

Now, let us consider only the P–R effect and the radial solar wind effect. We put $\gamma_T = 0$ into equations (69) and (70), and we obtain the following equation from these equations:

$$\frac{dp_\beta}{de_\beta} = \frac{p_\beta}{e_\beta}\left[-2\left(1+\frac{\eta_0}{\bar{Q}'_{pr}}\right)\beta\frac{\mu}{c}+\frac{1}{1-\beta}\frac{A}{R^2}K\right] \times \left[-\frac{5}{2}\left(1+\frac{\eta_0}{\bar{Q}'_{pr}}\right)\beta\frac{\mu}{c}+\frac{1}{1-\beta}\frac{A}{R^2}K\right]^{-1}. \quad (71)$$

Equation (71) yields the relation

$$p_\beta = p_{\beta \mathrm{in}}\left(\frac{e_\beta}{e_{\beta \mathrm{in}}}\right)^{4/5}, \qquad K \equiv 0, \quad (72)$$

where $p_{\beta \mathrm{in}}$ and $e_{\beta \mathrm{in}}$ are the initial values of the semilatus rectum and the eccentricity of the particle's orbit, respectively. Equation (72) can be considered as a generalization of the result obtained by Wyatt & Whipple (1950); we have taken into account not only the P–R effect, but also the radial solar wind effect. Equation (72) allows us to write the equation for the secular evolution of eccentricity in the form

$$\left\langle\frac{de_\beta}{dt}\right\rangle = -\frac{5}{2}\left(1+\frac{\eta_0}{\bar{Q}'_{pr}}\right)\beta\frac{\mu}{c}\frac{e_{\beta \mathrm{in}}^{8/5}}{p_{\beta \mathrm{in}}^2}\frac{\left(1-e_\beta^2\right)^{3/2}}{e_\beta^{3/5}}, \quad (73)$$

$K \equiv 0$.

It is evident from equation (71) that, because of the P–R effect and the radial solar wind, the particle is spiralling inwards to the Sun, for $K \equiv 0$. The semimajor axis a_β and the eccentricity e_β of the particle's orbit converge to 0 (see equations 72 and 73, and the relation between a_β, p_β and e_β). Equation (73) can offer the spiralling time of the particle with initial orbital elements $a_{\beta \mathrm{in}}$ and $e_{\beta \mathrm{in}}$ into the orbit with orbital elements a_β and e_β. This time is given by the relation

$$\tau\left(e_{\beta \mathrm{in}}, e_\beta\right) = -\frac{2}{5}\left[\left(1+\frac{\eta_0}{\bar{Q}'_{pr}}\right)\beta\frac{\mu}{c}\right]^{-1} a_{\beta \mathrm{in}}^2 \times \frac{\left(1-e_{\beta \mathrm{in}}^2\right)^2}{e_{\beta \mathrm{in}}^{8/5}} I\left(e_{\beta \mathrm{in}}, e_\beta\right),$$

$$I\left(e_{\beta \mathrm{in}}, e_\beta\right) = \int_{e_{\beta \mathrm{in}}}^{e_\beta}\frac{x^{3/5}}{\left(1-x^2\right)^{3/2}}\,dx,$$

$$K \equiv 0. \quad (74)$$

The spiralling time of the particle into the Sun is then

$$\tau\left(e_{\beta \mathrm{in}}, 0\right) = -\frac{2}{5}\left[\left(1+\frac{\eta_0}{\bar{Q}'_{pr}}\right)\beta\frac{\mu}{c}\right]^{-1} a_{\beta \mathrm{in}}^2 \times \frac{\left(1-e_{\beta \mathrm{in}}^2\right)^2}{e_{\beta \mathrm{in}}^{8/5}} I\left(e_{\beta \mathrm{in}}, 0\right),$$

$$K \equiv 0. \quad (75)$$

Let us consider two particles characterized by the values β_1 and β_2. Moreover, let the particles have the same value of $\bar{Q}'_{pr}$. If we are interested in the times the particles remain within an interval of the semimajor axes (a_{lower}, a_{upper}), then equation (74) yields $\tau_1/\tau_2 = \beta_2/\beta_1$, if the initial values of the semimajor axes and eccentricities are equal for both particles. On the basis of equations (69) and (70) ($K \equiv 0$), this result can also be approximately generalized to the case of the constant non-radial solar wind effect, assuming β_1, $\beta_2 \ll 1$.

Regarding the secular evolution, assuming that the particle's radius does not decrease, we have to solve only one differential equation (equation 73); equation (72) immediately yields the semilatus rectum and we can easily obtain the semimajor axis $a_\beta = p_\beta/(1 - e_\beta^2)$.

5.6 Summary

Our analytical approach shows that the decrease of the particle's radius as a result of the solar wind abrasion (corpuscular sputtering) can generate an increase of the particle's semimajor axis and eccentricity. This result is consistent with the detailed numerical calculations presented by Kocifaj & Klačka (2008).

6 NUMERICAL RESULTS

In this section, we concentrate on the orbital evolution of the IDP under the action of solar electromagnetic and corpuscular radiation. Solar wind erosion is also taken into account. The results are based on our new approach presented in Sections 2–4 and are compared with the standard approach only when the radial solar wind with constant η is taken into account.

6.1 Radial solar wind and particle erosion

Fig. 1 depicts the evolutions of the semimajor axis, eccentricity, argument of perihelion and particle radius for two particles under the action of the Sun's gravitational field, the P–R effect and constant radial solar wind. Evolutions are obtained from the numerical solutions of equations (58) and (56) with $\delta \equiv 0$. The values of the orbital elements are calculated using the central acceleration $-\mu(1-\beta)\mathbf{e}_R/r^2$. This is denoted by the subscript β in Fig. 1. The particles have an initial radius $R = 3$ μm, $\bar{Q}'_{pr} = 1$ and mass density $\varrho = 2000$ kg m^{-3}. The initial conditions for both particles are identical, $a_{\beta \mathrm{in}} = 14$ au, $e_{\beta \mathrm{in}} = 0.5$, $\omega_{\beta \mathrm{in}} = 90°$ and $f_{\beta \mathrm{in}} = 0$. The particles differ only in the value of K. The particle whose evolution is depicted by a solid line has $K = 2 \times 10^{-11}$ m au^2 yr^{-1}, which corresponds to a silicate (Mukai et al. 2001). The particle whose evolution is depicted by a dashed grey line has $K = 0$. For the first particle, it is assumed that $B = 0$ in equation (63). This means that $\bar{Q}'_{pr}$ is independent of the particle's radius. This is a good approximation for larger dust particles (see van de Hulst 1981). The evolutions of all orbital elements in Fig. 1 for these two different particles are practically identical. Therefore, the effect of solar erosion does not have a large influence on the evolution of orbital elements of μm-sized silicate dust particles. Fig. 2 shows the eccentricity of the dust particle as a function of the particle's radius for two cases. The solid black line corresponds to results of the numerical solution depicted in Fig. 1. The dashed grey line corresponds to values obtained from equation (67). The numerical and analytical approaches have a very good agreement.

6.2 Non-radial solar wind

If we take into account an even more realistic description of the solar wind, when its non-radial velocity component is considered, the resulting orbital evolution differs from the cases discussed in Section 6.1. Fig. 3 depicts two orbital evolutions of a dust particle with $\beta = 0.1$, $\bar{Q}'_{pr} = 1$ and $K = 0$ under the action of the solar wind. The black line is used for the non-radial solar wind ($\gamma_T = 0.05$, $\eta = 0.38$ and $u = 450$ km s^{-1} in equation 55 with the neglected solar wind pressure term) and the grey line is used for the radial solar wind (equation 58). The initial conditions for both evolutions are

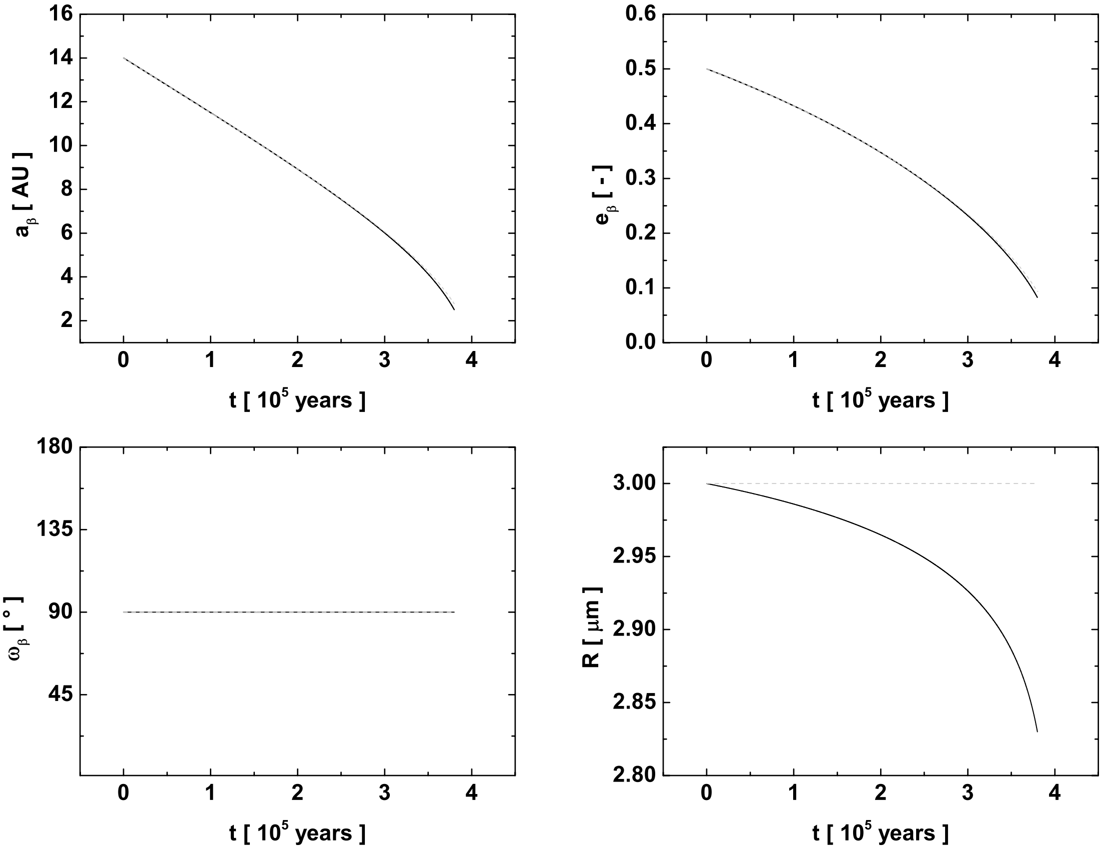

Figure 1. Evolutions of the semimajor axis, eccentricity, argument of perihelion and particle radius for two dust particles with initial radius $R = 3$ μm, $\bar{Q}'_{\mathrm{pr}} = 1$ and mass density $\varrho = 2000$ kg m^{-3}. In order to demonstrate the effect of solar wind erosion, we used particles with two different values of K. The first particle has $K = 2 \times 10^{-11}$ m au^2 yr^{-1} (solid black line) and the second has $K = 0$ (dashed grey line). The dependence of β on radius in equation (63) is characterized by the condition $B = 0$. The second particle has a constant radius. The figures show that the effect of solar wind erosion has a negligible influence on the time evolution of the particle's orbital elements.

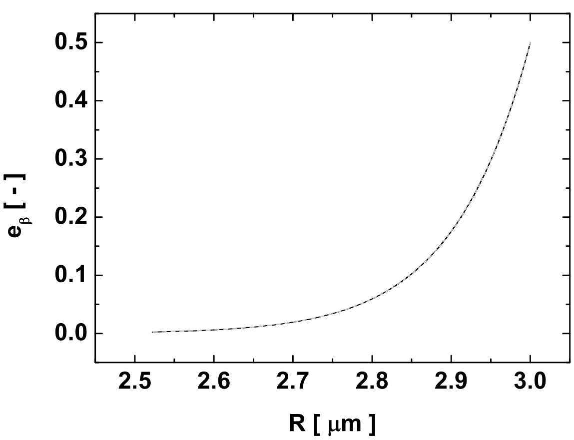

Figure 2. Dependence of orbital eccentricity on the particle's radius for $K = 2 \times 10^{-11}$ m au^2 yr^{-1}. The solid black line corresponds to the numerical solution presented in Fig. 1. The dashed grey line is obtained from equation (67). The results obtained with these two different methods are in good agreement (i.e. the lines overlap).

identical: $a_{\beta\mathrm{in}} = 27$ au, $e_{\beta\mathrm{in}} = 0.2$, $\omega_{\beta\mathrm{in}} = 0$ and $f_{\beta\mathrm{in}} = 0$. We can see that the secular semimajor axis can also be an increasing function under the action of the non-radial solar wind. This can easily be understood using equations (64). The secular time derivative of the semimajor axis can also be positive for $\gamma_{\mathrm{T}} > 0$. If

$$\left[\frac{1 + \eta_0/\bar{Q}'_{\mathrm{pr}}}{2\gamma_{\mathrm{T}} \left(\eta_0/\bar{Q}'_{\mathrm{pr}} \right) u_0} \right]^2 \mu(1 - \beta) < a_\beta \frac{1 - e_\beta^2}{\left(2 + 3e_\beta^2\right)^2}, \tag{76}$$

then the secular semimajor axis is an increasing function of time. The left-hand side of this inequality is a constant and the function $(1 - e_\beta^2)/(2 + 3e_\beta^2)^2$ has maximal value for $e_\beta = 0$. Hence, the condition for the minimal semimajor axis $a_{\beta\mathrm{min}}$, for which the secular time derivative of semimajor axis can be non-negative, is

$$a_{\beta\,\mathrm{min}} = \left[\frac{1 + \eta_0/\bar{Q}'_{\mathrm{pr}}}{\gamma_{\mathrm{T}} \left(\eta_0/\bar{Q}'_{\mathrm{pr}} \right) u_0} \right]^2 \mu(1 - \beta). \tag{77}$$

The secular semimajor axis is always a decreasing function of time when $a_\beta < a_{\beta\mathrm{min}}$. For the dust particle with $\beta = 0.1$ and $\bar{Q}'_{\mathrm{pr}} = 1$ and for the non-radial solar wind with $\gamma_{\mathrm{T}} = 0.05$, $\eta_0 = 0.38$ and $u = 450$ km s^{-1}, we obtain $a_{\beta\mathrm{min}} \approx 20.8$ au. If we use the parameters of the numerical solution depicted in Fig. 3 by the black line, from equation (64) we obtain $\langle \mathrm{d}a_\beta/\mathrm{d}t \rangle \approx 3.82 \times 10^{-7}$ au yr^{-1}, $\langle \mathrm{d}e_\beta/\mathrm{d}t \rangle \approx -4.67 \times 10^{-8}$ yr^{-1} and $\langle \mathrm{d}\omega_\beta/\mathrm{d}t \rangle \approx -3.63 \times 10^{-8}$ deg yr^{-1}. These values are in accordance with the values obtained from the numerical solution depicted in Fig. 3.

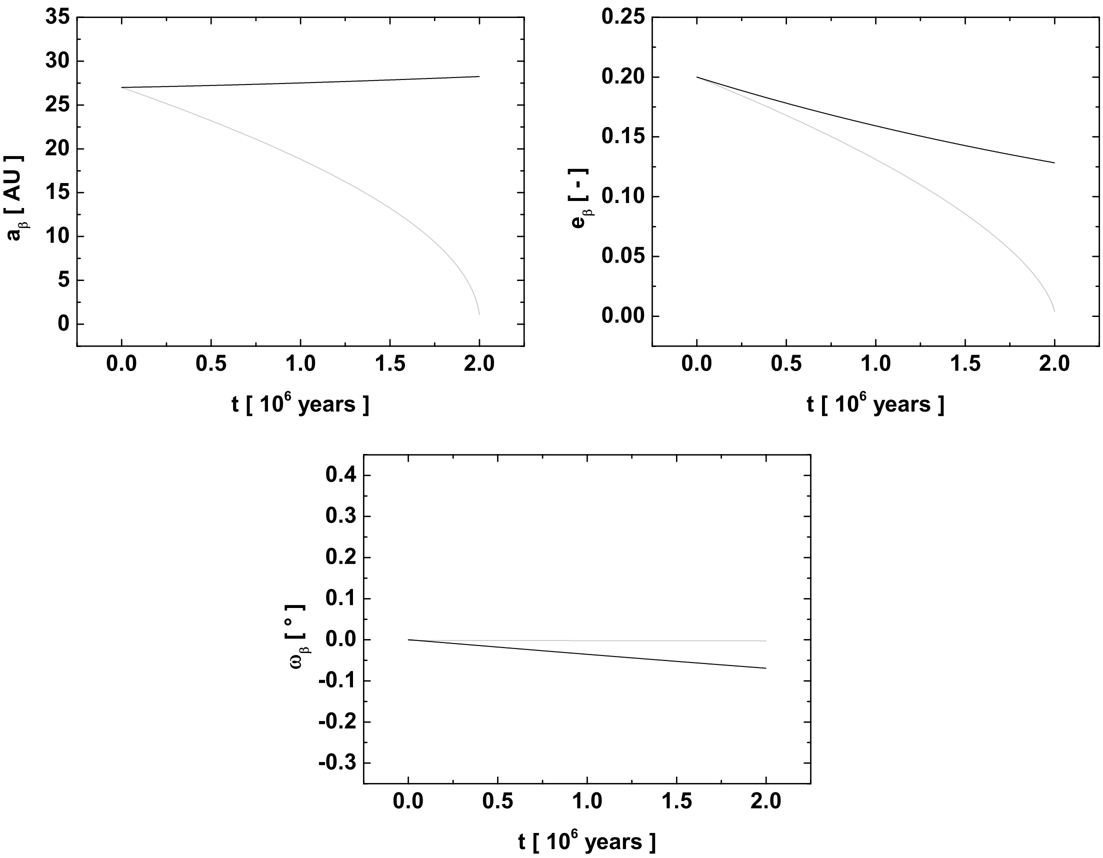

Figure 3. Evolutions of the semimajor axis, eccentricity and argument of perihelion for a dust particle with $\beta = 0.1$ and $\bar{Q}'_{\mathrm{pr}} = 1$ under the action of the non-radial solar wind (black line) and the radial solar wind (grey line). The semimajor axis can also be an increasing function of time when the non-radial component of solar wind is taken into account.

6.3 (Time-)variable solar wind

The equation of motion for a dust particle in the plane of the solar equator, represented by equation (55), also holds for the time-dependent solar wind. In this section, we generalize the results obtained in the previous two sections for a variable solar wind. In order to find the influence of the variability of the solar wind on the evolution of a particle's orbits, we have numerically solved equations (53) and (55). The results are depicted in Fig. 4. We have used the dust particle with $\beta = 0.1$, $\bar{Q}'_{\mathrm{pr}} = 1$ and $K = 0$. To visualize the differences, we have used four different equations of motion. The solid black line is used for the numerical solution of equations (53) and (55) with $\gamma_{\mathrm{T}} = 0.05$ and with the solar wind pressure term included. The dashed black line is used for the constant non-radial solar wind described by equation (55) with the solar wind pressure term neglected. The solid grey line is used for the variable radial solar wind ($\gamma_{\mathrm{T}} = 0$) described by equations (53) and (55) with the solar wind pressure term included; the second-order terms in v/u are neglected. Finally, the dashed grey line is used for the constant radial solar wind described by equation (58). The initial conditions for all four evolutions are identical: $a_{\beta\mathrm{in}} = 7$ au, $e_{\beta\mathrm{in}} = 0.5$, $\omega_{\beta\mathrm{in}} = 0$ and $f_{\beta\mathrm{in}} = 0$. Fig. 4 shows that the evolutions of a_β and e_β are slower for non-radial wind than for radial wind. This is because the non-radial component of the solar wind velocity vector accelerates the dust particle in the case of the prograde motion and $\gamma_{\mathrm{T}} > 0$. If we compare the evolutions for a given type of solar wind (i.e. radial or non-radial), we can see that the variability of solar wind does not have a large influence on the evolution of the semimajor axis and eccentricity. However, this is not true for the evolution of the argument of perihelion. We concentrate on this topic in the following section.

6.4 Resonances between particle's orbital period and solar cycle period

Fig. 4 also depicts the evolution of the argument of perihelion for a variable solar wind. It shows that the secular decrease of the argument of perihelion, theoretically consistent with equation (64) (see the evolutions under the action of the constant solar wind in Fig. 4), is characterized by abrupt changes. We can easily verify that the semimajor axes during these changes correspond to the semimajor axes for which the ratio of the particle's orbital period P and the solar cycle period T is equal to the ratio of two small natural numbers. The semimajor axis of the dust particle in these orbits can be calculated from

$$a_\beta = \left[\frac{\mu(1-\beta)}{4\pi^2} \right]^{1/3} \left(T \frac{p_{\mathrm{r}}}{q_{\mathrm{r}}} \right)^{2/3} , \tag{78}$$

where p_{r} and q_{r} are two natural numbers not equal to zero. When the ratio of two periods corresponds to a ratio of two small natural numbers, these processes are well known in astrophysics as resonances. For an exterior resonance $p_{\mathrm{r}}/q_{\mathrm{r}} > 1$, and we can write $p_{\mathrm{r}}/q_{\mathrm{r}} = (j_{\mathrm{r}} + s_{\mathrm{r}})/j_{\mathrm{r}}$, where j_{r} and s_{r} are two natural numbers, called the resonant number and the resonant order, respectively. For an interior resonance $p_{\mathrm{r}}/q_{\mathrm{r}} < 1$, and we can write $p_{\mathrm{r}}/q_{\mathrm{r}} = j_{\mathrm{r}}/(j_{\mathrm{r}} + s_{\mathrm{r}})$. For the special case of a 1/1 resonance, the resonant order is equal to zero.

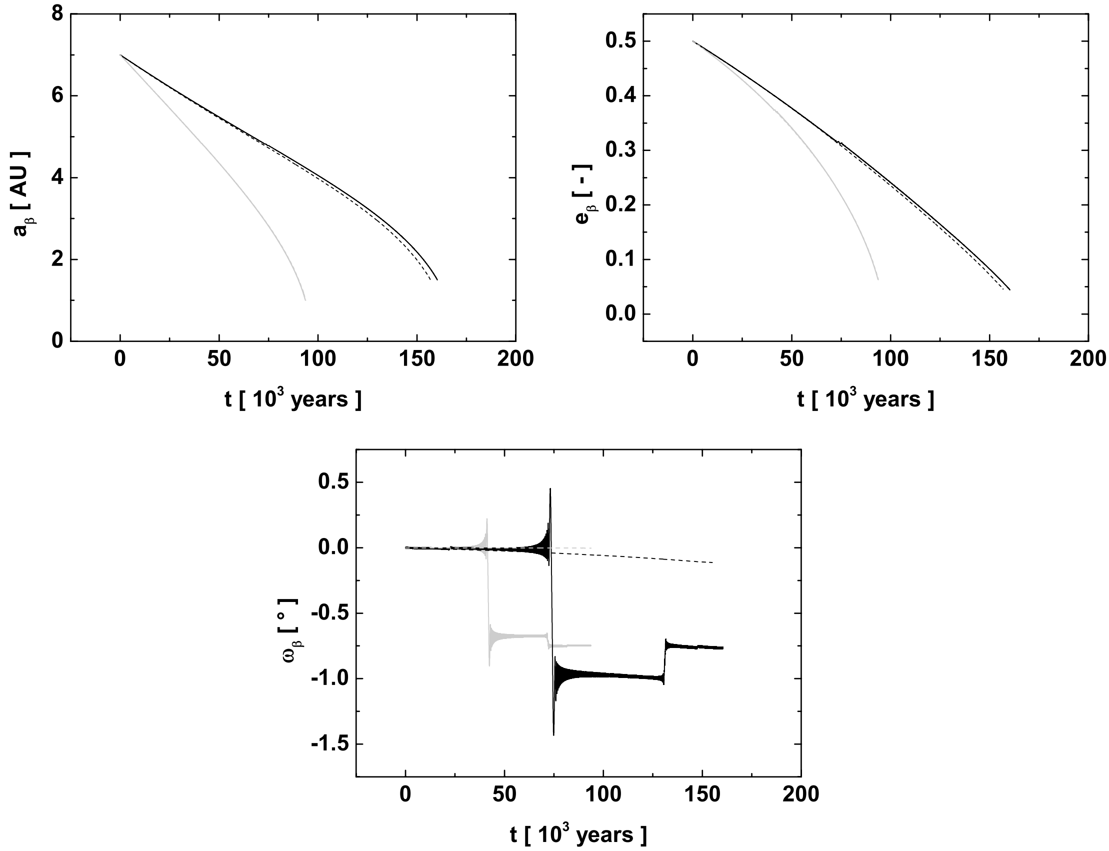

Figure 4. Evolutions of the semimajor axis, eccentricity and argument of perihelion for a dust particle with $\beta = 0.1$, $\bar{Q}'_{\rm pr} = 1$ and $K = 0$. Four different equations of motion are solved for equal initial conditions. The solid black line is used for the (time-)variable non-radial solar wind. The dashed black line is used for the constant non-radial solar wind. The solid grey line is used for the variable radial solar wind. Finally, the dashed grey line is used for the constant radial solar wind.

We can identify four resonances in Fig. 4, in both evolutions with the variable solar wind. The resonances are $p_{\rm r}/q_{\rm r} \in \{3/2, 1/1, 1/2, 1/3\}$. The resonances can also have an influence on the evolution of eccentricity. Such a situation is depicted in Fig. 5. We have used the dust particle with $\beta = 0.01$, $\bar{Q}'_{\rm pr} = 1$ and $K = 0$. The initial conditions are $a_{\beta{\rm in}} = a_{1/1\ 0.01} + 0.15$ au, $e_{\beta{\rm in}} = 0.1$, $\omega_{\beta{\rm in}} = 0$ and $f_{\beta{\rm in}} = 0$. Here, $a_{1/1\ 0.01}$ is the semimajor axis of the dust particle's orbit calculated from equation (78) for $p_{\rm r}/q_{\rm r} = 1/1$ and $\beta = 0.01$. The resonance between the orbital period and the solar cycle period does not have such a stabilization effect on the evolution of the semimajor axis as it occurs in the mean-motion resonance with a planet.

In the conventional approach, the time-independent radial solar wind is considered. Its action on a spherical IDP in the gravitational field of the Sun and solar electromagnetic radiation is characterized by a secular decrease of the particle's semimajor axis and eccentricity, and there is no shift of the perihelion of the particle. However, the more realistic situation differs from the conventional approach.

6.5 Spiralling times for dust grains inside the Earth's orbit

In the preceding sections, we have the presented possible cases of orbital evolutions under the action of a variable non-radial solar wind. However, an orbital evolution can be characterized by more relevant quantities. The speed of inspiralling towards the Sun is among these. We wish to find the relevance of our physical solar wind model compared with the conventional model, as for the evolution of an IDP. We deal with the following situation. A set of spherical dust particles was numerically integrated in order to obtain their evolution under the defined initial values of the orbital elements. The particles were initially ejected from a parent body with a semimajor axis $a_{\rm in} = 1$ au and eccentricity $e_{\rm in} \in \{0, 0.2, 0.4, 0.6, 0.8\}$ with zero ejection velocity. The motions in the plane of the solar equator were considered. The time variability of the solar wind does not play a significant role in the inner part of the Solar system. When the evolution of a dust particle with a semimajor axis $a_\beta < 1$ au is considered, the time variability can be neglected. However, the non-radial component of the solar wind velocity vector partially enhances the spiralling time of the particle toward the Sun. The detailed numerical calculations are summarized in Tables 1 and 2, for two values of $\bar{Q}'_{\rm pr}$ (e.g. 1 and 1/2). The gravity of the Sun, the P–R effect ($\beta = 0.01$) and the time-independent solar wind (radial and non-radial, $\gamma_{\rm T} = 0$ and 0.05) are considered. Tables 1 and 2 show that the real inspiralling time is greater than the time corresponding to the radial solar wind. The difference is greater than 21 per cent ($\tau_{\rm non-rad}/\tau_{\rm rad} = 1.212$) for $\bar{Q}'_{\rm pr} = 1$ and $e_{\rm in} = 0$, or even greater than 38 per cent ($\tau_{\rm non-rad}/\tau_{\rm rad} = 1.383$) for $\bar{Q}'_{\rm pr} = 1/2$ and $e_{\rm in} = 0$. Qualitatively, this result is immediately evident from equations (64), if $K \equiv 0$. The results presented in Tables 1 and 2 show that the ratio $\tau_{\rm non-rad}/\tau_{\rm rad}$ is a decreasing function of eccentricity: $\tau_{\rm non-rad}/\tau_{\rm rad} = 1.212$ for $e_{\rm in} = 0.0$ and $\tau_{\rm non-rad}/\tau_{\rm rad} = 1.108$ for $e_{\rm in} = 0.8$, if $\bar{Q}'_{\rm pr} = 1$, or $\tau_{\rm non-rad}/\tau_{\rm rad} = 1.383$ for $e_{\rm in} = 0.0$ and $\tau_{\rm non-rad}/\tau_{\rm rad} = 1.207$ for $e_{\rm in} = 0.8$, if $\bar{Q}'_{\rm pr} = 1/2$ and $\beta = 0.01$. This can easily be understood. Equation (64) ($K \equiv 0$) shows that the decrease of the semimajor axis

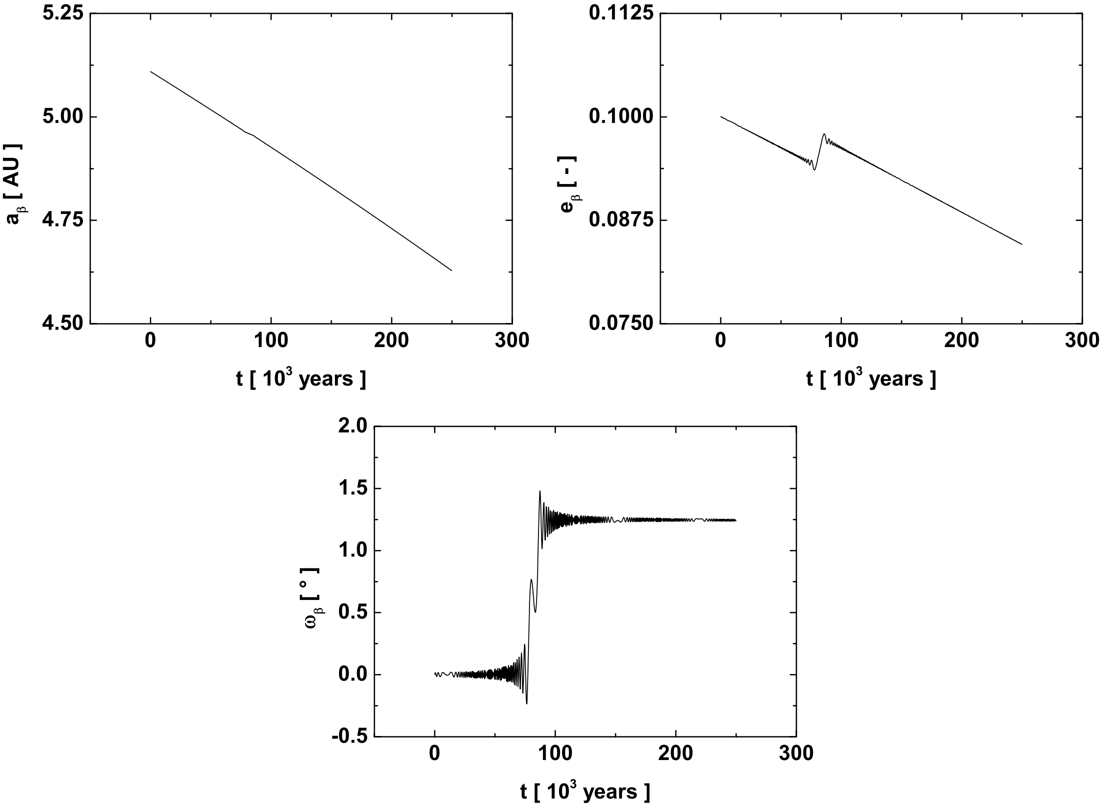

Figure 5. Evolutions of the semimajor axis, eccentricity and argument of perihelion for a dust particle with $\beta = 0.01$, $\bar{Q}'_{\rm pr} = 1$ and $K = 0$ under the action of the variable non-radial solar wind. The variability of the solar wind can also change the evolution of eccentricity during the resonances.

Table 1. Spiralling time of a dust grain towards the Sun ejected from the perihelion of a parent body with the semimajor axis $a_{\rm in} = 1$ au and various eccentricities $e_{\rm in}$. The ejection velocity equals zero. The gravity of the Sun, the P–R effect ($\beta = 0.01$, $\bar{Q}'_{\rm pr} = 1$) and the time-independent solar wind (radial and non-radial, $\gamma_{\rm T} = 0$ and 0.05) are considered. The variability of the solar wind with the solar cycle does not change the presented results.

$e_{\rm in}$	Spiralling time (10^4 yr)	
	PR+SW(R)	PR+SW(NR)
0.0	3.00	3.54
0.2	2.83	3.33
0.4	2.37	2.76
0.6	1.64	1.87
0.8	0.75	0.82

Table 2. Spiralling time of a dust grain towards the Sun ejected from the perihelion of a parent body with the semimajor axis $a_{\rm in} = 1$ au and various eccentricities $e_{\rm in}$. The ejection velocity equals zero. The gravity of the Sun, the P–R effect ($\beta = 0.01$, $\bar{Q}'_{\rm pr} = 1/2$) and the time-independent solar wind (radial and non-radial, $\gamma_{\rm T} = 0$ and 0.05) are considered. The variability of the solar wind with the solar cycle does not change the presented results.

$e_{\rm in}$	Spiralling time (10^4 yr)	
	PR+SW(R)	PR+SW(NR)
0.0	2.30	3.18
0.2	2.18	2.99
0.4	1.82	2.44
0.6	1.27	1.63
0.8	0.58	0.70

is more rapid for greater values of eccentricity, and moreover the influence of the solar wind's non-radial component, with respect to the radial component, is more important for lower values of particle eccentricity. The effect of the non-radial component of the solar wind is more important for smaller values of particle eccentricity (i.e. surely in the vicinity of the Sun, but the spiralling towards the Sun is very rapid for this zone because the corresponding values of the particle's semimajor axis are very small; see equation 64, mainly da_β/dt).

7 DISCUSSION

We have derived the relativistically covariant equation of motion for the action of solar wind corpuscles on the motion of an IDP. Regarding the spherical shape of the particles, the equation of motion is represented by equation (30). It differs from the force conventionally presented in the literature (although only to the first order in $\boldsymbol{v}/u$):

$$\boldsymbol{F}_{\rm sw} = F'_{\rm sw}\left[\left(1-\frac{2\dot{r}}{u}\right)\boldsymbol{e}_{\rm R} - \frac{r\dot{\theta}}{u}\boldsymbol{e}_{\rm T}\right].$$

Here, $\boldsymbol{v}$ is the velocity of the grain $\boldsymbol{v} = \dot{r}\boldsymbol{e}_{\rm R} + r\dot{\theta}\boldsymbol{e}_{\rm T}$, u is the heliocentric solar wind speed and $F'_{\rm sw}$ is the force on the dust for $\boldsymbol{v} = 0$ (Minato et al. 2004, Section 2.1; the equivalent force is presented in equation (7.10) of Mann 2009; see also Burns, Lamy & Soter 1979, p. 12).

We have to stress that the standard form corresponds to equation (30), if we use the physically incorrect identity $\sigma'_{\rm tot} = \sigma'_{\rm pr}$ (see Appendix A) and $x' = 1$ (the reality is $1 < x' < 3$, approximately). Equation (30) yields the correct limiting result $u \to c$ equivalent to the P–R effect.

Regarding the practical application of the physical results discussed above, equations (53)–(56) are astronomically relevant. The decrease of the mass of the particle can cause the particle to spiral outwards from the Sun and not towards the Sun, as is commonly accepted for the solar wind action on an IDP. This result is evident from the analytical equations presented in Section 5 (e.g. equation 59), and detailed numerical calculations confirming this result can be found in Kocifaj & Klačka (2008).

In Section 6, we concentrated on the action of the solar wind on the motion of an IDP for cases when the variable flux of the solar wind energy and the non-radial solar wind velocity are considered. Particle erosion does not have a large influence on the orbital evolution of μm-sized dust particles, if they are not in the close vicinity of the Sun, if they do not consist of very volatile materials and if the efficiency factor of radiation pressure $\bar{Q}'_{\rm pr}$ of the particle does not strongly depend on the particle's radius. It is generally believed that the shift of perihelion does not exist, for either the solar wind effect or the P–R effect. However, the real action of the solar wind differs from the action of the P–R effect: the P–R effect really produces no shift of perihelion. Both constant and variable non-radial solar wind can produce a shift of perihelion. However, if the dust particle is not in the close vicinity of the Sun, the shift of perihelion is small, and in general it can be neglected. The non-radial solar wind velocity can lead to outspiralling from the Sun, in the region of the outer planets. The particle might or might not spiral towards the Sun because of the simultaneous action of the P–R effect and solar wind effect (see also Klačka et al. 2008). We have also found the existence of resonances between the orbital period of the particle and the solar cycle period. The constant non-radial solar wind can also yield results that are different from the conventional approach (constant radial wind), in particular for the speed at which the particle spirals towards the Sun. The effect of the non-radial component of the solar wind on dust particle motion does not play an important role in the inner parts of the Solar system. Inside the Earth's orbit, the non-radial component of the wind enhances the spiralling time towards the Sun by less than 40 per cent in comparison with the radial solar wind (if $\bar{Q}'_{\rm pr}$ is greater than 1/2).

8 CONCLUSION

We derive the relativistically covariant equation of motion for an arbitrarily shaped dust particle under the action of solar wind. The change of the particle's mass is an indispensable part of the space–time formulation of the equation of motion for the action of the solar wind. The solar wind effect would reduce to the P–R effect, in the limiting case when (i) the solar wind speed tends to the speed of light, (ii) there is no decrease in the mass of the IDP and (iii) the velocity of the solar wind is radial. However, the solar wind can have a qualitatively different effect on the orbital evolution of an IDP, because points (ii) and (iii) are not fulfilled, in general.

The secular evolution is given by equations (63)–(65) for the time-independent solar wind. The decrease in the mass of the IDP and the non-radial component of the solar wind velocity can cause the particle to spiral outwards from the Sun. The solar wind (non-radial or radial accurate to the second order in v/u) leads to the shift of perihelion of the particle. The shift of perihelion caused by the solar wind in the Solar system is low, but non-zero.

If we consider solar wind variability with an 11.1-yr solar cycle, there are resonances between the orbital period of the dust particle and the solar cycle period. If the particle's semimajor axis is close to the resonant values, then the evolution of eccentricity and the argument of perihelion can be significantly affected.

Our results could have important consequences for the orbital evolution of dust belts/discs in the vicinity of stars with stellar winds.

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APPENDIX A: TOTAL SCATTERING CROSS-SECTION FOR A SPHERICAL DUST PARTICLE

We use some sort of approximation to the hard-core scattering problem, which corresponds to the limiting case of a short-range potential $V(r) = \infty$ for $r < R$, $V(r) = 0$ for $r > R$ (hard-core potential; e.g. Iro 2002, p. 158). We consider the scattering of point solar wind corpuscles from an almost hard sphere of radius R. In the case of the infinitely hard sphere of radius R, 'the dynamics reduces to the laws of reflection at the surface of the sphere' (Iro 2002, p. 158). The result of classical physics is the following: 'In the case of a finite-range potential, the total [scattering] cross section is finite and gives the effective area of the potential. (This is actually the definition of a finite-range potential.) For example, when point masses are incident onto a hard sphere, $\sigma'_{\rm tot}$ is the cross-section of the sphere – only particles incident within that area are deflected.' (Iro 2002, p. 161).

However, the correct physics for the incident electromagnetic radiation suggests that the geometric cross-section might not lead to correct results for the incident solar wind corpuscles (Klačka 2008a,b). Inspired by de Broglie's idea about the wave characteristic of massive particles, we can conclude that scattering by a hard sphere at very high energies leads to the total scattering cross-section

$$\sigma'_{\rm tot} = 2\pi R^2, \tag{A1}$$

and that 'the classical total cross-section is just half of the quantum-mechanical result in the limit of very short wavelength' (Messiah 1999, pp. 393–395).

If we use some sort of approximation to the hard sphere, we can use the total scattering cross-section given by equation (A1) in our paper.

Regarding the comparison of the results obtained by quantum (subscript 'q') and non-quantum (subscript 'nq') physics, we use equation (26) or equation (30). The non-quantum approach uses $\sigma'_{\rm tot} = \sigma'_{\rm pr} = A' = \pi R^2$:

$$\left(\frac{{\rm d}p^\mu}{{\rm d}\tau}\right)_{\rm nq} = \frac{1}{c} A' S \frac{\alpha\omega}{u}\left(\xi^\mu - x' \frac{U^\mu}{c}\right). \tag{A2}$$

A comparison between equations (30) and (A2) yields (comparing coefficients at ξ^μ and U^μ):

$$\sigma'_{\rm pr} = A', \qquad (x')_q = \left[(x')_{\rm nq} - 1\right]\frac{\sigma'_{\rm pr}}{\sigma'_{\rm tot}} + 1. \tag{A3}$$

Also, using equation (A1), we obtain

$$(x')_q = \frac{1}{2}\left[(x')_{nq} + 1\right]. \tag{A4}$$

The case $\left\{\sigma'_{\rm pr} = A', \sigma'_{\rm tot} = 2A'\right\}$ is analogous to the cases of perfectly absorbing or reflecting spheres within the geometrical optics approximation for electromagnetic radiation (Klačka 2008b). This analogy also explains the importance of quantum physics in our derivations – non-quantum physics would not yield correct results in the limit $u \to c$.

APPENDIX B: EMISSION FROM THE PARTICLE

The other possible force influencing the dynamics of a dust particle might originate from an emission (e.g. radioactive decay). Let the particle emit an energy $E'_{\rm em}$ per unit time due to the emission in its proper reference frame. We suppose that this emission is represented by the flux of corpuscules with a speed $u'_{\rm em}$. Furthermore, we declare the orthonormal vector basis $\{\boldsymbol{f'}_j; j = 1, 2, 3\}$, as used in Section 2.2. The corresponding velocities are $\boldsymbol{u'}_{{\rm em},j} = u'_{\rm em}\boldsymbol{f'}_j$, $j = 1, 2, 3$.

The outgoing four-momentum of the emission per unit time, in the proper reference frame of the particle, is

$$p'^\mu_{\rm em} = \left(\frac{1}{c}E'_{\rm em}; \frac{1}{c}E'_{\rm em}\sum_{j=1}^{3} r'_j \frac{\boldsymbol{u'}_{{\rm em},j}}{c}\right), \tag{B1}$$

where r'_j ($j = 1, 2, 3$) are dimensionless coefficients expressing the part of the total flux of radiation that is emitted in the corresponding directions.

The Lorentz transformations of equation (A1) yield the following outgoing four-momentum per unit time in the stationary reference frame:

$$p^\mu_{\rm em} = \frac{1}{c}E'_{\rm em}\frac{U^\mu}{c} + \frac{1}{c}E'_{\rm em}\sum_{j=1}^{3} r'_j\left(\xi^\mu_{{\rm em},j} - \frac{U^\mu}{c}\right), \tag{B2}$$

where

$$
\begin{aligned}
\xi^{\mu}_{\mathrm{em},\,j} &= \left(\frac{1}{\omega_{\mathrm{em},\,j}} ; \frac{1}{\omega_{\mathrm{em},\,j}} \frac{\boldsymbol{u}_{\mathrm{em},\,j}}{c} \right) , \\
\omega_{\mathrm{em},\,j} &\equiv \gamma\,(v) \left(1 - \frac{\boldsymbol{v} \cdot \boldsymbol{u}_{\mathrm{em},\,j}}{c^2} \right) , \\
\boldsymbol{u}_{\mathrm{em},\,j} &= \left[\gamma\,(v) \left(1 + \frac{\boldsymbol{v} \cdot \boldsymbol{u}'_{\mathrm{em},\,j}}{c^2} \right) \right]^{-1} \\
&\quad \times \left[\boldsymbol{u}'_{\mathrm{em},\,j} + \left\{ [\gamma(v) - 1] \frac{\boldsymbol{v} \cdot \boldsymbol{u}'_{\mathrm{em},\,j}}{v^2} + \gamma\,(v) \right\} \boldsymbol{v} \right] , \\
j &= 1, 2, 3.
\end{aligned}
\tag{B3}
$$

Thus, the expression on the right-hand side of equation (A2) should be added to the right-hand side of equation (22) when we would also like to take into account the effect of the emission from the dust particle.

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