

# The past and future motion of Comet P/Swift–Tuttle

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## ABSTRACT

The orbit of P/Swift–Tuttle is investigated by way of a long-term integration forward to AD 2392 and backward to 703 BC. Two of its previous returns prior to the telescopic period, in AD 188 and 69 BC, are identified in Chinese records. No other observations of P/Swift–Tuttle have been found. The non-gravitational forces that affect the motion of most active comets appear to be negligible for this comet, suggesting that either the comet outgasses radially toward the Sun and in a symmetric fashion about perihelion and/or it is far more massive than periodic comets of comparable activity (e.g. P/Halley). Our successful integration is consistent with all the observed returns of the comet. The unobserved returns between AD 188 and 1737 are easily explained, as the comet did not approach the Earth closely enough to allow naked-eye visibility. Our integration and the observing conditions at each return suggest that the comet has maintained about the same intrinsic brightness for more than two millennia. The lack of non-gravitational effects for this comet and the relative constancy of its intrinsic brightness place constraints upon the lifetime of its active area(s) and its spin state. Our prediction of the comet's return in 2126 places it well away from the Earth's position at the nodal crossing.

**Key words:** methods: numerical – celestial mechanics, stellar dynamics – comets: individual: P/Swift–Tuttle.

## 1 INTRODUCTION

P/Swift–Tuttle, the comet associated with the Perseid meteor stream, was successfully recovered on 1992 September 26 by Tsuruhiko Kiuchi (IAU Circ. 5620), after a lapse of 130 years since its last return in 1862. Until this recovery, it was not clear when, or even if, the comet would be found because, at the time, only the 1862 observations were clearly associated with Comet Swift–Tuttle. The first detailed analysis on the orbit of P/Swift–Tuttle was carried out by Hayn (1889), based on the 1862 observations which he grouped into seven normal places. Marsden (1973) utilized the individual observations as given by Hayn, and re-computed the orbit using 212 observations over the interval 1862 July 22–October 22. Marsden, whose orbit was consistent with Hayn's earlier result, estimated a two-year uncertainty in the orbital period, which he determined to be 119.98 yr. Using the fact that the comet was not obviously observed in the past, Marsden predicted, from a solution using the 1862 data alone, that P/Swift–Tuttle would return sometime between 1980 and 1983. He searched backwards in time for any possible 18th century appearance of this comet, and established that a comet observed in 1737 by the Jesuit missionary Koeigler in Beijing was a possible sighting

of P/Swift–Tuttle, as was also suggested earlier by Lynn (1902). Since the observation interval in 1737 was very short (1737 July 3–10), Marsden found that five different orbital solutions were possible, and that one of the solutions could be linked to the 1862 observations to give reasonable residuals if a large and negative radial component of the non-gravitational accelerations was employed. He then predicted that P/Swift–Tuttle would return in 1992 November, if the comet was not seen in the early 1980s.

The recovery of P/Swift–Tuttle in 1992 September not only showed that the second prediction by Marsden (1973) was correct, but also that the comet observed in 1737 was truly the same comet making one of its earlier appearances. With three consecutive observed returns, the comet's long-term motion could be investigated, including a prediction for the comet's next return to perihelion in the first half of the 22nd century. Marsden (1992a) noticed that the observational data in 1862, apart from the October data, could be linked reasonably well with the available pre-perihelion data in 1992, if large non-gravitational effects were included. However, the resulting predicted perihelion passage time for the 1737 return was 15 months too late. Marsden (1992a) published an orbit that did not take non-gravitational effects into account, but fitted the observations in 1992, and in

1862 October very well (though ignoring all other 1862 data), and also the perihelion passage time in 1737 to within one day. The extrapolation of this orbit gave a future time of perihelion passage of 2126 July 11. Due to the large uncertainty concerning the role of non-gravitational effects, Marsden noted that a change in the perihelion date by +15 d could cause the comet to strike the Earth on 2126 August 14.

Problems with various orbital solutions arose, because the only observations available in 1862 October are from the Cape of Good Hope and these are not consistent with the remaining 1862 observations. Either these data are accurate and large, short-term non-gravitational forces were affecting the comet's motion about this time, or the 1862 October data are discordant. By eliminating the 1862 October data altogether, Yeomans (1992) successfully linked the 1737, 1862 and 1992 data without the need for non-gravitational effects.

In order to study the long-term motion of P/Swift-Tuttle and to understand the role of any non-gravitational effects that might be operative, we have conducted a search for earlier returns of the comet. In Section 2, we give details of an orbit determination that takes into account the 1737, 1862 and 1992–93 data. In Section 3, we use the determined orbits to initialize backward integrations to 703 BC and, in Section 4, we identify the previous returns of the comet from the Chinese sources. In Section 5, we present a discussion of our results and predict the comet's future returns.

## 2 INITIAL CONDITIONS FOR LONG-TERM INTEGRATION

The orbit determination program is a linearized, weighted least-squares estimation algorithm, in which observational data are used to improve an existing orbit, and the dynamic model includes all planetary perturbations at each time-step. The numerical integrator uses a variable-step, variable-order Adams method (Krogh 1972), and the step-size varies to ensure that the estimated local error at each step is less than an input tolerance of  $10^{-13}$  au d<sup>-1</sup>. The partial derivatives necessary for adjusting the initial conditions are integrated along with the object's equations of motion. The basic form of Cowell's equations of motion is

$$\ddot{\mathbf{r}} = -k^2 \frac{\mathbf{r}}{r^3} + \mathbf{f}_p, \quad (1)$$

where  $\ddot{\mathbf{r}}(\dot{x}, \dot{y}, \dot{z})$  is the acceleration of the comet in units of au d<sup>-2</sup>,  $k$  is the Gaussian gravitational constant, and  $\mathbf{r}(x, y, z)$  and  $r=|\mathbf{r}|$  are the heliocentric position and distance of the comet, respectively. The first term on the right-hand side of equation (1) is the direct attraction due to the central primary mass, the Sun. Here the mass of the Sun is taken to be unity. The second term on the right-hand side is the planetary perturbational acceleration, given by

$$\mathbf{f}_p = k^2 \sum_j m_j \left( \frac{\mathbf{r}_j - \mathbf{r}}{\rho_j^3} - \frac{\mathbf{r}_j}{r_j^3} \right), \quad (2)$$

where  $\mathbf{r}_j(x_j, y_j, z_j)$  are positions of all of the perturbing masses  $m_j$  expressed in terms of the solar mass, from Mercury to Pluto,  $\rho_j = |\mathbf{r}_j - \mathbf{r}|$  are the distances between the comet and

the planets, and  $r_j = |\mathbf{r}_j|$  are the planetary distances from the centre of the Sun.

For short-period comets, we have to take into account the non-gravitational accelerations due to the rocket-like effects of outgassing volatiles from the icy nucleus. For consistency with the planetary ephemeris, we also have to consider effects from post-Newtonian, relativistic orbital perturbations. These accelerations introduce two other terms into equation (1):

$$\ddot{\mathbf{r}} = -k^2 \frac{\mathbf{r}}{r^3} + \mathbf{f}_p + \mathbf{f}_n + \mathbf{f}_r. \quad (3)$$

The non-gravitational term is given (Marsden, Sekanina & Yeomans 1973)

$$\mathbf{f}_n = (A_1 \hat{\mathbf{r}} + A_2 \hat{\mathbf{T}}) \left\{ \alpha \left( \frac{r}{r_0} \right)^{-m} \left[ 1 + \left( \frac{r}{r_0} \right)^n \right]^{-l} \right\}, \quad (4)$$

where  $\hat{\mathbf{r}} = \mathbf{r}/r$  is the radial unit vector defined outward along the Sun–comet radius vector, and  $\hat{\mathbf{T}} = (\mathbf{r}\dot{\mathbf{r}} - \dot{\mathbf{r}}\mathbf{r})/h$  is the transverse unit vector directed in the direction of the comet's motion and normal to  $\hat{\mathbf{r}}$  in the orbital plane. In the transverse unit vector equation,  $\dot{\mathbf{r}}(\dot{x}, \dot{y}, \dot{z})$  represents the velocity of the comet,  $\dot{\mathbf{r}} = \mathbf{r} \cdot \dot{\mathbf{r}}/r$  and  $h^2 = (y\dot{z} - z\dot{y})^2 + (z\dot{x} - x\dot{z})^2 + (x\dot{y} - y\dot{x})^2$ . The parameters  $A_1$  and  $A_2$  are, respectively, the magnitude of the radial and transverse acceleration components at 1 au. These non-gravitational parameters ( $A_1, A_2$ ) are determined along with the orbital elements, and they can be determined fairly accurately if three apparitions of the comet are available.  $\alpha = 0.111\,262$  is a normalizing constant such that the term within the large brackets in equation (4) becomes unity at 1 au. This model of the non-gravitational accelerations is based upon the sublimation from a water-ice nucleus (Marsden et al. 1973). The values obtained for water ice are  $r_0 = 2.808$  au with the components  $m, n$  and  $l$  equal to 2.15, 5.093 and 4.6142, respectively.

The term  $\mathbf{f}_r$  in equation (3) is due to relativistic effects. The parametrized post-Newtonian equations based on those derived by Anderson et al. (1975) can be written for a comet as

$$\mathbf{f}_r = \frac{k^2}{c^2 r^3} \left[ 4k^2 \frac{\mathbf{r}}{r} - (\dot{\mathbf{r}} \cdot \dot{\mathbf{r}}) \mathbf{r} + 4(\mathbf{r} \cdot \dot{\mathbf{r}}) \dot{\mathbf{r}} \right], \quad (5)$$

where  $c$  is in units of au d<sup>-1</sup>.

Two orbit determinations were computed using the available observations and planetary ephemeris DE200 (Standish 1990). These orbits are given in Table 1. The first orbit utilized 594 observations over the interval from 1862 July 23 to 1993 March 15. The 1862 October data from the Cape of Good Hope are not included in our orbital solutions for reasons given in Section 4. No non-gravitational effects were employed for the first orbit. For the second orbit, an additional eight observations from 1737 were used. All of the observations from 1862 and 1992–93 retained for orbital solutions were weighted equally with noise values of 1 arcsec. The eight observations from 1737 were given noise values of 15 arcmin, as they are approximate positions only. For the second orbit, a solution was also made for the radial and transverse non-gravitational parameters ( $A_1, A_2$ ). Although the 1737 residuals reflect the fact that these are

**Table 1.** Orbital solutions for Comet P/Swift–Tuttle.

	Orbit 1	Orbit 2
Observation Interval	1862 July 23–1993 Mar 15	1737 July 2–1993 March 15
Number of Observations	594	602
RMS Residual	1.37''	1.38''
Perihelion Time, $T$	1992 Dec 12.3239437	1992 Dec 12.3239546
Perihelion Distance, $q$	0.958217507	0.958217427
Eccentricity, $e$	0.963589236	0.963589498
Arg. of Perihelion, $\omega$	153.0013797	153.0013980
Long. Asc. Node, $\Omega$	139.4444214	139.4444208
Inclination, $i$	113.4265824	113.4265816
Epoch	1992 Dec 4.0 (TDB)	1992 Dec 4.0 (TDB)
$A_1 \times 10^{-8}$	0	-0.0118 ( $\pm 0.0452$ )
$A_2 \times 10^{-8}$	0	-0.00155 ( $\pm 0.0002$ )

only approximate observations, no systematic residual trends remain in the 1862 and 1992–93 data. For the second orbit, the rms residual is a weighted value.

### 3 THE LONG-TERM INTEGRATION

For the long-term integration of the comet's motion backward in time, a somewhat different integration process was used. In part, this was due to the absence of suitable planetary coordinates earlier than AD 1600 from planetary ephemeris DE200. The coordinates of the planets for the long-term integration are taken directly from the JPL ephemeris DE102, which covers the interval from 1411 BC to AD 3002. Although this is an older ephemeris, it is the only one that covers the entire period of the historical astronomical observations. None of the newer ephemerides produced at JPL has been integrated for more than a few centuries in time. However, we can apply corrections to the coordinates of DE102 to make it approximate closely the more recent ephemeris DE118 (Newhall, Standish & Williams 1983). Since both DE102 and DE118 are referenced to the 1950 system, and the current investigation employs the J2000 system, it was necessary to make a transformation to J2000 by using the rotation matrix given by Standish (1982). This is the same matrix as was actually used to create DE200 from DE118. The masses of the planets used in the integration were taken from the DE200 ephemeris.

The integrator itself uses a predictor-corrector procedure based on the Gauss–Jackson method (for details see Herrick 1972). A central-difference approach up to the 6th differences is employed in the present investigation. The components of the total acceleration acting on the comet due to the Sun and planets, the non-gravitational effects, and relativity are integrated step by step. The fixed step-size chosen for the integration is 0.25 d. This step-size gives a difference of the order of  $10^{-17}$  au d $^{-2}$  in the 6th differences in our integration scheme. One can choose a smaller step-size, but this is not necessary since we are working with double-precision Fortran.

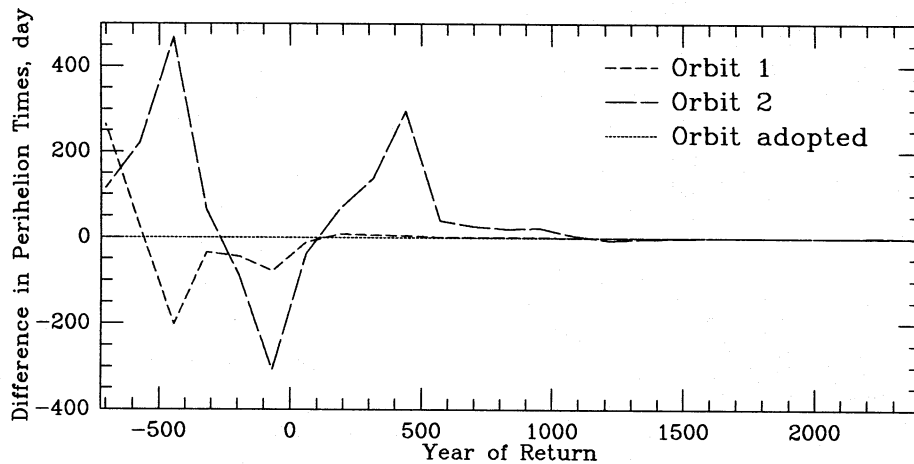
Beginning with orbit 1 in Table 1, the comet's motion was integrated backwards in time. The computed times of perihelion passage during the observed returns in AD 188 and 69 BC were compared with the 'observed' times of perihelion

determined directly from the Chinese observations (see Section 4). Since the initial integration predicted a perihelion time that was 4 d early in AD 188 and 79 d late in 69 BC, we made a series of very small adjustments in the initial value of the orbital eccentricity until the observations in AD 188 and 69 BC were well represented.

The optimal eccentricity adjustment was  $+1.57 \times 10^{-9}$ , a correction that is about a factor of 6 less than the formal rms uncertainty of the eccentricity itself. Since we held the initial perihelion distance fixed, our correction introduced an effective change,  $\Delta a$ , to the orbital semimajor axis ( $a = 26.31687451$  au) of  $1.13 \times 10^{-6}$  au or only 170 km. Although the eccentricity correction to the initial orbital elements,  $\Delta e$ , is less than the uncertainty associated with the eccentricity itself, the predicted times of perihelion passage in AD 188 and 69 BC are very sensitive to the selected value of  $\Delta e$ . As a result of the close Earth approach to within 0.129 au on AD 188 July 25 and an approach to Jupiter of 1.78 au on AD 57 November 18, the Chinese observations in 69 BC could not be matched unless the computed time of perihelion passage in 188 was July 10.5. This computed time of perihelion passage is far more tightly constrained than is possible from the Chinese observations themselves.

Fig. 1 shows the sensitivity of the predicted time of perihelion passage to the assumed eccentricity correction, and also the backward integration of orbit 2 in Table 1. This latter orbit, with non-gravitational effects operative, is not compatible with the observational data in AD 188 and 69 BC. The initial integration based on orbit 2 predicted a perihelion time that was 64 d early in AD 188 and 307 d late in 69 BC. Even by adjusting the eccentricity of orbit 2 in a similar manner to that carried out for orbit 1, we cannot match the observed perihelion times for these two returns.

At the end of each cometary revolution around the Sun, a summary of the integration results is printed out. This includes rectangular coordinates for the position and velocity, mean motion, and the osculating orbital elements. The summary also includes a table of minimum distances between the comet and the planets. A list of close approaches to the planets from Venus to Saturn for all returns between 703 BC and AD 2392 is shown in Table 2. During this period, there are no particularly close approaches to Uranus and Neptune. The closest encounters to these two planets were at 2.5 and 6.5 au, respectively.



**Figure 1.** The difference in the times of perihelion passage for all returns from 703 BC to AD 2392 between orbit 1 (solved without non-gravitational forces) and the final adopted orbit, and also between orbit 2 (with non-gravitational forces included) and the final adopted orbit.

**Table 2.** Minimum distances between comet Swift-Tuttle and the planets Venus, Earth, Mars, Jupiter and Saturn.

Venus Date	au	Earth Date	au	Mars Date	au	Jupiter Date	au	Saturn Date	au
2392/ 9/30	0.285	2392/ 9/17	0.501	2392/10/26	0.631	2392/ 5/20	4.454	2389/12/ 3	1.048
2261/ 9/ 6	0.363	2261/ 8/24	0.147	2261/ 7/22	1.228	2261/ 6/ 3	4.638	2261/ 8/22	8.803
2126/ 6/21	0.923	2126/ 8/ 5	0.153	2126/ 5/10	1.579	2126/ 9/22	4.359	2124/ 2/26	1.346
1992/12/ 4	0.679	1992/11/ 7	1.165	1992/10/15	1.568	1991/ 6/11	1.848	1993/ 1/19	8.797
1862/ 7/26	1.026	1862/ 8/30	0.342	1862/ 8/27	0.729	1861/ 1/17	2.030	1859/10/19	1.113
1737/ 7/11	0.360	1737/ 7/19	0.375	1737/ 3/ 4	1.598	1737/ 7/ 4	4.078	1736/11/14	7.916
1610/ 3/ 5	0.374	1609/12/ 7	1.369	1610/ 4/ 8	1.071	1609/ 9/10	4.288	1610/ 3/17	8.799
1479/10/17	0.537	1479/ 9/29	0.947	1479/ 7/ 1	1.594	1479/ 5/ 4	4.153	1476/11/ 3	1.425
1348/ 3/29	1.097	1348/ 6/17	0.716	1347/12/11	1.668	1348/ 2/12	4.629	1348/ 5/14	8.737
1212/10/28	0.711	1212/10/ 9	1.080	1212/ 6/28	1.631	1213/ 1/28	4.439	1210/ 8/13	1.847
1079/10/27	0.604	1079/ 9/ 9	0.734	1079/11/12	0.958	1078/ 2/14	2.027	1079/10/27	8.762
950/ 3/17	1.089	950/ 6/10	0.824	950/ 5/ 3	0.479	948/ 2/ 7	3.756	947/ 1/ 3	2.294
826/ 5/19	0.419	826/ 6/ 9	0.815	826/ 4/19	0.830	826/ 2/ 4	4.634	825/ 3/21	6.238
698/ 8/ 4	1.082	698/ 9/ 1	0.658	698/ 8/ 5	1.404	697/ 9/ 6	2.233	698/10/ 2	8.708
569/ 3/26	0.359	569/ 5/13	1.293	568/12/25	1.593	567/10/19	1.637	564/10/12	5.216
441/11/24	0.321	441/10/ 5	1.086	442/ 2/ 3	1.699	442/ 1/26	4.397	441/ 2/ 2	7.353
316/12/15	0.796	316/10/ 1	1.051	316/ 9/30	1.340	316/ 6/27	4.466	316/11/13	8.661
188/ 7/ 5	0.637	188/ 7/25	0.129	188/ 7/27	0.424	187/ 7/24	2.333	182/ 6/ 9	8.752
59/ 5/ 5	0.779	59/ 6/23	0.455	59/ 3/23	1.569	57/11/18	1.778	58/ 7/14	7.063
-68/ 8/12	0.816	-68/ 8/23	0.620	-68/ 9/21	0.327	-68/10/21	4.083	-68/ 9/24	8.682
-193/ 2/17	0.906	-193/ 5/16	1.163	-193/ 2/ 1	1.448	-194/ 9/27	4.183	-193/ 7/10	9.309
-321/ 9/25	0.577	-321/ 9/12	0.879	-321/ 6/23	1.588	-322/ 4/10	1.723	-323/12/13	3.434
-446/ 4/29	1.097	-446/ 7/ 1	0.247	-446/ 7/18	0.694	-446/ 6/28	4.006	-446/ 4/23	8.658
-573/ 7/16	0.830	-573/ 8/ 6	0.400	-573/ 5/14	1.612	-573/ 2/23	4.227	-573/10/ 3	8.950
-702/ 4/12	0.373	-702/ 5/27	0.877	-703/11/ 4	1.678	-703/ 6/29	3.088	-705/12/ 5	1.549

The results of the integration are given in Table 3. The respective columns are the year of return, the time of perihelion passage  $T$  (both calendar date and Julian date are given), eccentricity  $e$ , perihelion distance  $q$ , period  $P$ , argument of perihelion  $\omega$ , longitude of the ascending node  $\Omega$ , inclination  $i$ , and the epoch of osculation. The dates are given in the Julian Calendar prior to 1582 October 4 and in the Gregorian Calendar after 1582 October 15. We have adopted the convention that year 0 = 1 BC, -1 = 2 BC, and so on.

#### 4 IDENTIFICATION OF PAST RETURNS

The first step in identifying the previous returns of a comet is to search through the appropriate catalogue of observational records and cometary orbits (e.g. Ho 1962; Ho & Ang 1970; Marsden & Williams 1992) for any past observations. If a comet is found near the predicted time of perihelion passage, then an ephemeris is constructed from the osculating orbital elements. An initial long-term integration is usually performed to provide the necessary orbital elements. Next, the



**Table 3.** Osculating orbital elements for comet P/Swift–Tuttle (J2000.0).

Return			T(ET)		q(au)	e	P(Yrs)	$\omega$	$\Omega$	$i$	Epoch
2392	2392	9	16.11262	2594978.61262	0.9498238	0.9634832	132.66	153.12568	139.62064	113.58504	2392 9 17.0
2261	2261	8	10.73872	2547095.23872	0.9584214	0.9635052	134.59	153.13973	139.74166	113.49411	2261 8 11.0
2126	2126	7	12.41024	2497757.91024	0.9562718	0.9638758	136.20	153.11853	139.60915	113.40857	2126 7 15.0
1992t	1992	12	12.32394	2448968.82394	0.9582175	0.9635892	135.02	153.00138	139.44442	113.42658	1992 12 4.0
1862III	1862	8	23.42278	2401375.92278	0.9626545	0.9627979	131.68	152.77369	139.37148	113.56644	1862 8 8.0
1737II	1737	6	15.85369	2355652.35369	0.9799742	0.9613652	127.77	152.69309	139.46155	113.67987	1737 6 4.0
1610	1610	2	6.69101	2309137.19101	0.9766687	0.9620380	130.50	152.82010	139.50112	113.55631	1610 2 6.0
1479	1479	10	18.29315	2261552.79315	0.9698281	0.9628540	133.41	152.88476	139.55365	113.48837	1479 10 18.0
1348	1348	5	2.32867	2213536.82867	0.9752229	0.9629495	135.05	152.96871	139.66877	113.47667	1348 5 2.0
1212	1212	11	6.13635	2164050.63635	0.9737912	0.9632596	136.46	152.93792	139.54068	113.38788	1212 11 6.0
1079	1079	9	17.61023	2115422.11023	0.9718508	0.9629921	134.58	152.90036	139.40658	113.48602	1079 9 17.0
950	950	4	19.45072	2068153.95072	0.9762973	0.9620177	130.32	152.70299	139.30202	113.67368	950 4 19.0
826	826	4	19.49363	2022862.99363	0.9770292	0.9614039	127.37	152.86643	139.44021	113.93156	826 4 19.0
698	698	9	6.41835	1976250.91835	0.9726568	0.9620703	129.86	152.86503	139.42888	113.84648	698 9 6.0
569	569	3	1.08586	1928944.58586	0.9702814	0.9623991	131.09	152.70112	139.32056	113.78758	569 2 28.0
441	441	11	3.07052	1882439.57052	0.9793991	0.9615435	128.53	152.48903	139.23131	113.85888	441 11 2.0
316	316	10	27.22997	1836776.72997	0.9777498	0.9615439	128.21	152.68367	139.30258	113.89181	316 10 27.0
188	188	7	10.54528	1789916.04528	0.9722372	0.9621649	130.27	152.63962	139.25850	113.86404	188 7 10.0
59	59	5	17.98690	1742744.48690	0.9698231	0.9624007	131.00	152.52240	139.18612	113.87888	59 5 17.0
69 BC	–68	8	27.10172	1696459.60172	0.9808664	0.9613614	127.91	152.42151	139.12362	113.90174	–68 8 26.0
194 BC	–193	3	9.53874	1650632.03874	0.9775645	0.9615623	128.26	152.53233	139.11327	113.93442	–193 3 9.0
322 BC	–321	9	27.65803	1604082.15803	0.9710275	0.9619826	129.09	152.46289	139.03279	114.03470	–321 9 27.0
447 BC	–446	6	2.14601	1558308.64601	0.9833087	0.9611945	127.56	152.41816	139.03830	113.96947	–446 6 2.0
574 BC	–573	8	1.41288	1511981.91288	0.9794732	0.9617904	129.79	152.53682	139.06154	113.85545	–573 8 1.0
703 BC	–702	4	3.28685	1464744.78685	0.9729864	0.9624775	132.05	152.53402	139.08842	113.82023	–702 4 3.0

computed positions around the time of observations are plotted on a star map marked with the relevant constellations. The path of the comet can then be easily checked against the descriptions given by the records. In cases where the plotted path of the comet does not match the observations, it may be necessary to adjust the time of perihelion to obtain a best fit. For example, this procedure was required for Halley’s comet, especially for those returns with a very close approach to the Earth (Yeomans & Kiang 1981; Stephenson & Yau 1985).

The only observations of P/Swift–Tuttle known prior to 1862 are found in Chinese histories. We have checked both the Korean and Japanese histories for the relevant period, but there do not seem to be any observations from these two sources. Only three early apparitions of P/Swift–Tuttle before the 1862 return can be identified from the Chinese observations. They are the returns of AD 1737, AD 188 and 69 BC. In the following discussion of the past returns of Comet Swift–Tuttle, we assume that the comet’s apparent magnitude,  $m_v$ , is reasonably well represented by

$$m_v = 4.5 + 5 \log \Delta + 15 \log r, \quad (6)$$

where  $\Delta$  and  $r$  are the geocentric and heliocentric distances in au. Equation (6) is determined from observations during the 1992/3 apparition (Marsden 1992c). We discuss and justify this assumption in Section 5.

#### 4.1 1862 return

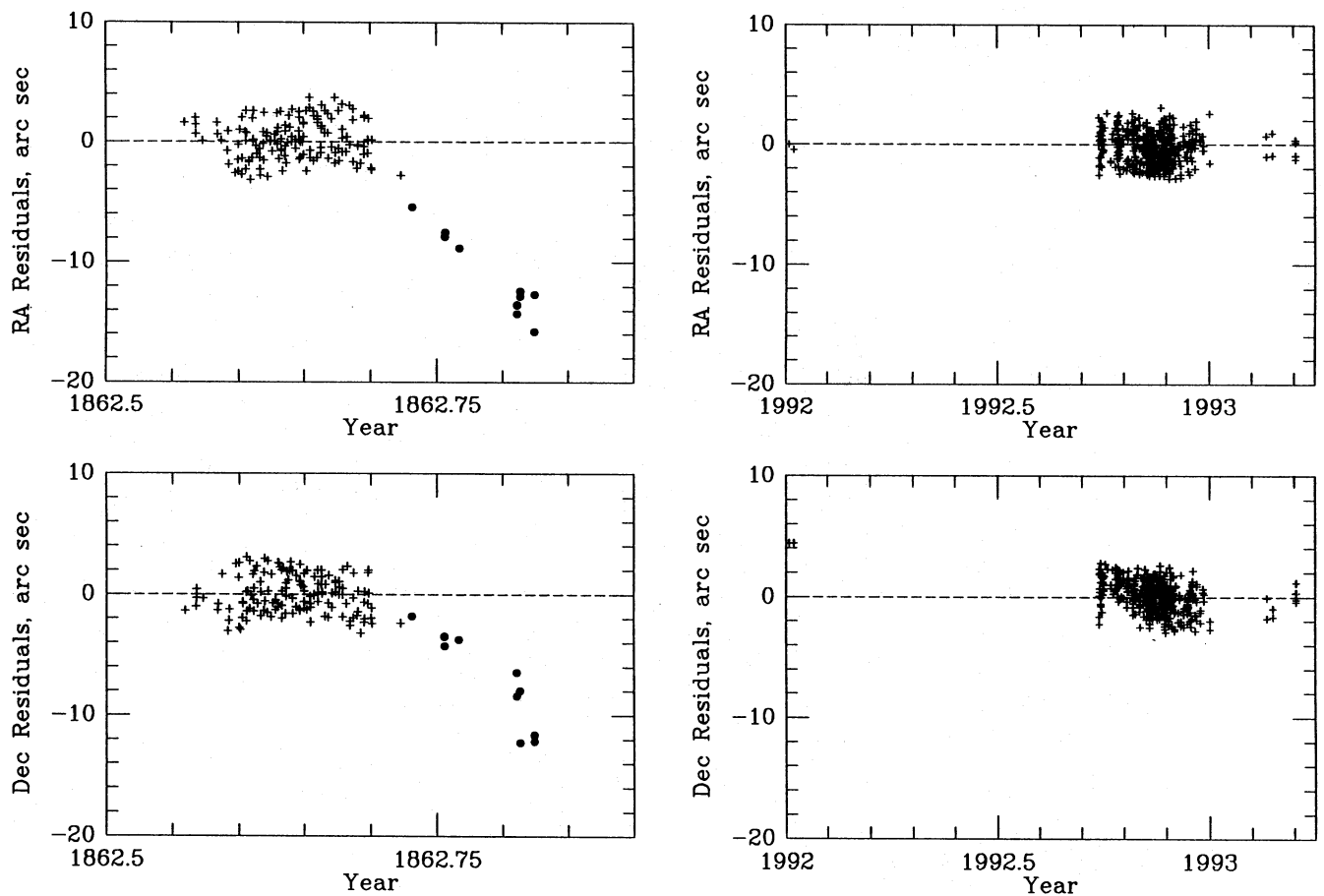
Details of Western observations during the comet’s return in 1862 can be found in Marsden (1973) and will not be dis-

cussed in detail here. Mention has already been made of the discordant 1862 October observations from the Cape of Good Hope, South Africa. Owing to the comet’s southern declinations in late September and October, the 10 available observations were all made at the Cape. These observations are discordant with the remaining 594 observations from 59 other observatories when one attempts to fit them with the traditional models for cometary dynamics. Either short-term, unmodelled perturbations in the comet’s motion moved the object off the computed path in 1862 late September and October (and subsequently returned it thereafter) or there are errors in these observations. We consider the latter option to be the more likely. There may be systematic errors in the micrometer offsets between the comet and neighbouring reference stars as measured at the Cape in 1862 late September and October. The observation residuals (observed minus computed) for orbit number 1 are given in Fig. 2. Although they were not used in the solution for either orbit 1 or 2, the 1862 Cape data residuals are shown for comparison. The Cape data are clearly discordant.

Chinese observations for the 1862 return are not critical in our discussion. A summary of Chinese observations is given in Table 4.

#### 4.2 1737 return

The only observations for this return were obtained by the Jesuit missionary Koegler in Beijing. Ignatius Koegler (1680–1746) was the 6th Jesuit Director of the Chinese Astronomical Bureau from 1720 to 1746 (Yau 1993). He was involved in initiating the building of a large equatorial



**Figure 2.** The orbital residuals (in arcsec) from our orbit number 1 are displayed here as a function of time for both the 1862 and 1992–93 apparitions. The upper and lower two plots display the right ascension and declination residuals, respectively. All nine observations from the Cape of Good Hope in 1862 October and one in 1862 September are denoted with filled circles; they were not used in our orbital solutions. These 10 Cape observations are clearly discordant with the remaining 594 observations from 59 other observatories.

armillary sphere in 1744, which can still be seen on the platform above the Old Beijing Observatory today. The Chinese record contained in chapter 39 of the *Ch'ing-shih Kao* (Drafted History of the Ch'ing Dynasty 1644–1911) states

Ch'ien-lung reign period 2nd year, 6th month, day *ting-mao* (JD 235 5674 = 1737 July 7). 'A strange star appeared to the east of Yu-keng. Its colour was white. It belonged to Lou (the 16th Lunar Mansion). It moved towards the south-west, and was still visible on day *ping-tzu* (1737 July 16):'

The path of P/Swift–Tuttle for 1737 is given in Fig. 3. The asterisks represent Koegler's observed positions of P/Swift–Tuttle during the 1737 return. The open squares represent the calculated positions based on the osculating elements for 1737 listed in Table 3. The crude positional determinations by Koegler can be seen from those asterisks which are not centred on the square markers. The record in *Ch'ing-shih Kao* appears to be only a summary of naked-eye observations carried out at that time. The descriptions generally conform to the path of the comet in Fig. 3. When it was first observed on July 7, to the east of the Yu-keng asterism,

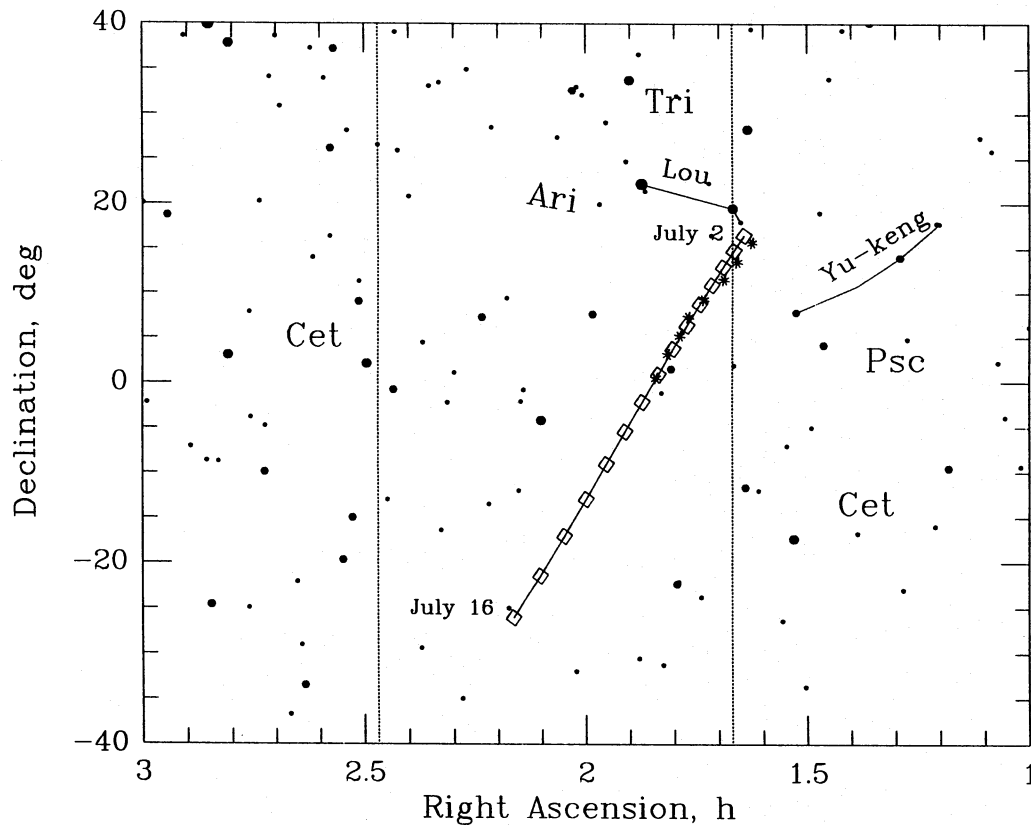
its apparent magnitude was 3.4. It is worth noting that the 69 BC return was also discovered with a similar magnitude (see below). It seems that magnitude 3.4 must represent a limit for the visibility of Comet Swift–Tuttle to the unaided eye.

The dotted lines represent the boundaries between the Lunar Mansions. These are a group of asterisms situated roughly along the celestial equator for the purpose of providing reference stars for the determination of right ascension. A detailed discussion on the Lunar Mansions and Chinese asterisms in general may be found in Kiang (1972). The record says that the comet was inside the Lunar Mansion Lou. It can be seen from Fig. 3 that from July 7 to July 16 Comet Swift–Tuttle was indeed located inside the Lunar Mansion Lou. The record mentions that the comet then moved towards the south-west instead of south-east. The confusion is most likely due to the ecliptic longitude being used rather than the right ascension. The Jesuits introduced the ecliptic system to China around the middle of the 17th century. On July 16, the last date of observation, the magnitude of the comet was still at its peak magnitude of 3.1. According to our computation, it should have been seen with the naked eye in the morning sky before sunrise in a near-south-easterly direction for a few more days until its heliacal setting on July 21.

Table 4. A summary of Chinese observations of comet P/Swift-Tuttle for the 1862 return.

No.	Date	Record
1.	1862 Aug 9	T'ung-chih reign period, 1st year, 7th month, 14th day. In the evening, there was a broom star with a length of more than a <i>chang</i> (more than ten degrees). It was pointing straight towards the wall of Tzu-wei. It lasted until August and then disappeared. ( <i>Yang-hsin Hsien-chih</i> , chap. 2)
2.	1862 Aug 10	T'ung-chih reign period, 1st year, 7th month, 15th day. (i) A broom star was seen at the Big Dipper. ( <i>Ts'en-hsiu Chu-ch'eng Hsien Hsu-chih</i> , chap. 1) (ii) A broom star was seen. It lasted until mid-August and then extinguished. ( <i>Hsu-hsiu Chu-yeh Hsien-chih</i> , chap. 1) (iii) A broom star was seen at the north-west. ( <i>Wu-yang Chih-yu</i> , chap. 5)
3.	1862 Aug 18	T'ung-chih reign period, 1st year, 7th month, 23rd day. At night, a broom star appeared in the north-western direction. ( <i>Luan-ch'eng Hsien-chih</i> , chap. 3)
4.	1862 Aug 19	T'ung-chih reign period, 1st year, 7th month, 24th day. (i) At night, a long star was seen at the north. Its color was bluish-white like smoke. It was five to six <i>ch'ih</i> (five to six degrees) in length. It was pointing down towards the south. On the night of the 7th day of the 8th month (Aug 31), it was seen in the west, and it set towards the east. ( <i>Nao-p'ing Hsien-chih</i> , chap. 13) (ii) A broom star was seen at the wall of the Tzu-wei Enclosure. Its vapour was about a <i>chang</i> (about ten degrees) in length. It disappeared after more than a month. ( <i>Wen-shui Hsien-chih</i> , chap. 1) (iii) A broom star rose from the north-west. Its length was several <i>chang</i> (several tens of degrees). It was pointing straight towards the wall of the Tzu-wei Enclosure. In the 8th month (Aug 25–Sept 23), it gradually moved into T'ien-lao and extinguished. ( <i>Hui-min Hsien-chih</i> , chap. 17)
5.	1862 Aug 20	T'ung-chih reign period, 1st year, 7th month, 25th day. (i) A broom star was seen at the north-west. ( <i>Ch'ing-shih-kao</i> , chap. 21) (ii) At night, a broom star was seen in the western direction. ( <i>Hsu-hsiu Shensi-Sheng T'ung-chih-kao</i> , chap. 201)
6.	1862 Aug 20 and 21	T'ung-chih reign period, 1st year, 7th month, 25th and 26th day. During the night, a broom star was seen at the north-west. ( <i>Ch'ing-ch'ao Hsu Wen-hsien T'ung-k'ao</i> , chap. 303)
7.	1862 Aug 21	T'ung-chih reign period, 1st year, 7th month, 26th day. At night, a broom star was seen at the Big Dipper. ( <i>Hsu-yung T'ing-chih</i> , chap. 51)
8.	1862 Aug 21 and 22	T'ung-chih reign period, 1st year, 7th month, 26th and 27th day. At night, a broom star was seen at the north-west. ( <i>Chekiang Hsu T'ung-chih-kao</i> , chap. 1)
9.	1862 Aug 23	T'ung-chih reign period, 1st year, 7th month, 28th day. (i) A broom star was seen. Its rays were straight. It disappeared on the 11th day of the 9th month (Oct 4). ( <i>Wu-yuan Hsien-chih</i> , chap. 64) (ii) A broom star was seen. ( <i>K'ai-hua Hsien-chih</i> , chap. 14)

Remarks. These records are contained in the local histories of various regions of China. They have been grouped according to the date of observations. Only records with a date accurate to the day are included in this list. Tzu-wei is an asterism with two walls forming an enclosure which occupies about 25° of the North Celestial Pole. T'ien-lao is an asterism in UMa, west of the Big Dipper.



**Figure 3.** The computed daily motion of Comet Swift-Tuttle (open squares) from July 2.7 to July 16.7 in 1737 and the corresponding observed positions by Kogler (asterisks) from July 2.7 to July 9.7.

#### 4.3 188 return

No observations of a comet that could be P/Swift-Tuttle are found in the interval from 1737 to 188. The 188 return is contained in chapter 20 of the *Hou Han-shu* (History of the Later Han Dynasty AD 25–220).

Chung-p'ing reign period 5th year, 6th month, day *ting-mao* (JD 178 9934 = 188 July 28). 'A guest-star as large as a vessel with a capacity of three pints appeared at Kuan-so. It moved south-west and entered the T'ien-shih (Enclosure). It reached Wei (the 6th Lunar Mansion) and then disappeared.'

The time of perihelion passage deduced from the description, taking July 28 as the date the comet appeared at Kuan-so, is one day on either side of 188 July 15, and thus consistent with the date of about July 15 suggested by Kronk (1992). The path of P/Swift-Tuttle for the 188 return, as given by our integration, is shown in Fig. 4. The path indicates that P/Swift-Tuttle appeared at Kuan-so on July 25. On the date given by the record, July 28, the comet would have already passed through the T'ien-shih Enclosure. The record seems to be giving a retrospective account. On July 28, the magnitude of the comet was 0.5, already past its peak brightness of 0.1 mag. Taking 3.4 mag as our discovery limit for P/Swift-Tuttle, the comet would have become visible to the naked eye on July 7. It is curious that there are no records earlier than July 28. Perhaps the original detailed observations were condensed by the compilers of the history. The description 'as large as a vessel' could mean a nebulous

appearance without a tail. Although the record mentions no other date, it is likely that it would have been visible until at least mid-August, when it would be in the region near the Lunar Mansion Wei.

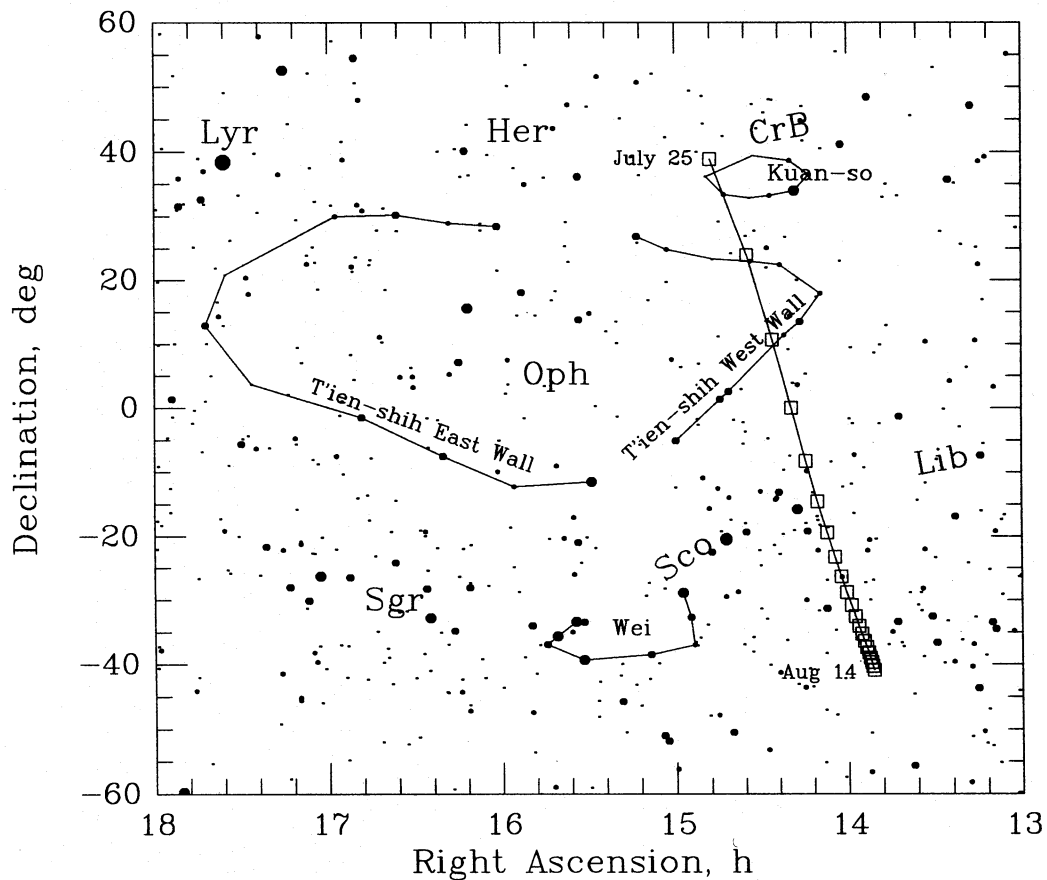
#### 4.4 69 BC return

The observations for the 69 BC return of Comet Swift-Tuttle may be found in chapter 26 of the *Ch'ien Han-shu* (History of the Former Han Dynasty 206 BC–AD 8). The record gives the following details concerning the observations of P/Swift-Tuttle.

Ti-chieh reign period 1st year, 6th month, day *ping-yin* (JD 169 6453 = BC 69 August 20). 'Another guest-star was seen to the north-east of Kuan-so. It moved in a southerly direction. On the night of *kuei-yu* (BC 69 August 27) it entered the T'ien-shih (Enclosure). Its rays were pointing south-east. Its colour was white.'

The time of perihelion passage obtained from the Chinese description is about August 27 in 69 BC, a result that is consistent with the suggestions by Hasegawa (1979) and Kronk (1992) that the time of perihelion passage was 69 BC August 26–28. The path of P/Swift-Tuttle for 69 BC, as given by our integration, is depicted in Fig. 5. On the first date (August 20) given by the record, which is the likely discovery date, the comet attained a magnitude of 3.4. On this date, Comet Swift-Tuttle was at the north-east of the asterism Kuan-so, as described in the record. It then moved south. On the night of August 27, it entered the enclosure of the





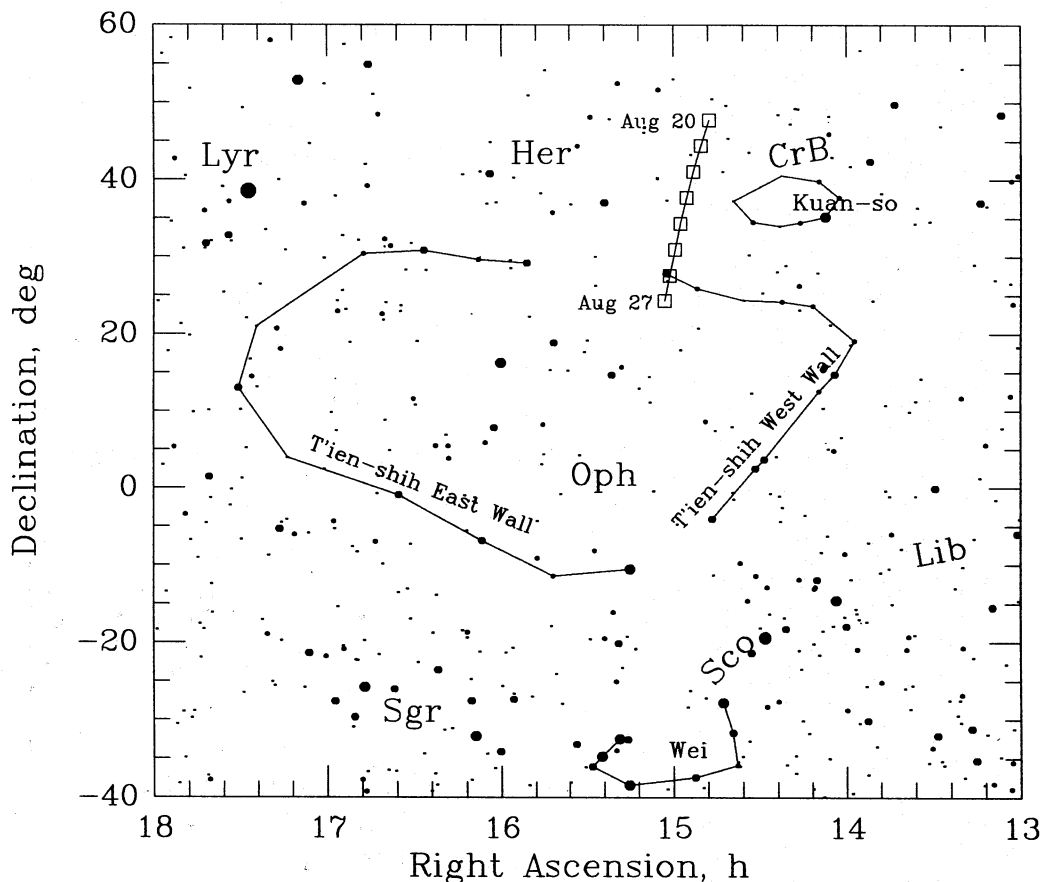
**Figure 4.** The computed daily motion of P/Swift-Tuttle (open squares) as seen at 8 pm (local time) from July 25 to August 14 during the AD 188 return.

asterism T'ien-shih. Its apparent magnitude was again 3.4, having reached its peak magnitude of 3.3 on August 24. Although the record mentions that its tail was pointing south-east, it does not give an estimate of its length. It is possible that the tail was too short to be noted. The tail was described as white, suggesting that a dust tail was being described, rather than a bluish ion tail. According to our calculations, the comet would have been visible in the early part of September, but no observations are known after August 27.

Apart from the above identified returns, there are three other returns of P/Swift-Tuttle which were in the borderline of visibility. They are the returns of AD 1079, 698 and 59. At the 1079 return, Comet Swift-Tuttle reached a peak magnitude of 3.7, and the conditions for its observation were favourable. Its position after dark was some  $55^\circ$  in altitude in a near-west direction. However, at this magnitude it did not attract the attention of any observers at the time. The 698 return is a little brighter, with a peak magnitude of 3.4. At the time it reached its maximum magnitude, it was about  $56^\circ$  above the western horizon in the evening sky. It stayed at this magnitude for four nights, yet there were no reports of its observation. The computed magnitude for the 698 return seems to suggest that this apparition must have come very close to being discovered. Perhaps it was missed merely due to adverse weather conditions. The 59 return attained a maximum magnitude of about 3.6. By the time it reached the peak magnitude, Comet Swift-Tuttle was already close to the

horizon, at an altitude of  $19^\circ$  at the beginning of morning twilight. It would have been seen in the south-east in the morning before sunrise. It was visible at this magnitude for only a few days before its heliacal setting. Even if the comet was at its usual discovery magnitude of 3.4, it might still have been missed due to its unfavourable location in the morning sky where the denser atmosphere near the horizon could easily obscure it from view.

If our numerical extrapolation of the comet's motion is correct prior to the 69 BC apparition, the comet would have been visible to the naked eye at the end of June in 447 BC with a peak magnitude of 2.1, and in August 574 BC with a magnitude of 2.4. However, no records of these returns can be located from the Chinese sources. It is likely that the relevant Chinese observations during these two apparitions have not been preserved. In fact, very few records of cometary observations, and astronomical phenomena in general, have survived before about 200 BC. This circumstance is possibly due to the infamous 'Burning of the Books' at the command of the first emperor of China, Ch'in Shih-huang, in 213 BC and the subsequent sacking of his capital Hsien-yang, where the official archive was kept, in 206 BC (Twitchett & Loewe 1986). Astronomical records that survived from the period before 200 BC tend to be very brief. For example, the record of the earliest known apparition of Comet Halley in 240 BC is so poor that it was not regarded as a positive sighting of the comet until the discovery of its 164 BC return on Babylonian cuneiform text tablets (Stephenson, Yau &



**Figure 5.** The computed daily motion of P/Swift-Tuttle (open squares) as seen at 8 pm (local time) from August 20 to August 27 during the 69 BC return.

Hunger 1985). Unless further material is unearthed, it seems that there is not much hope of Comet Swift-Tuttle having a longer history than its brighter counterpart Halley.

Without constraints from early observations, a continuation of the integration backwards for any substantial length of time has relatively little value. For the present investigation, we have continued optimistically until the return of 703 BC. The close approach to Jupiter of 1.72 au on 323 BC April 10, and a subsequent close approach to the Earth of 0.247 au, on 447 BC July 1, might have perturbed the motion of Comet Swift-Tuttle to such an extent that its orbit becomes unreliable beyond 447 BC.

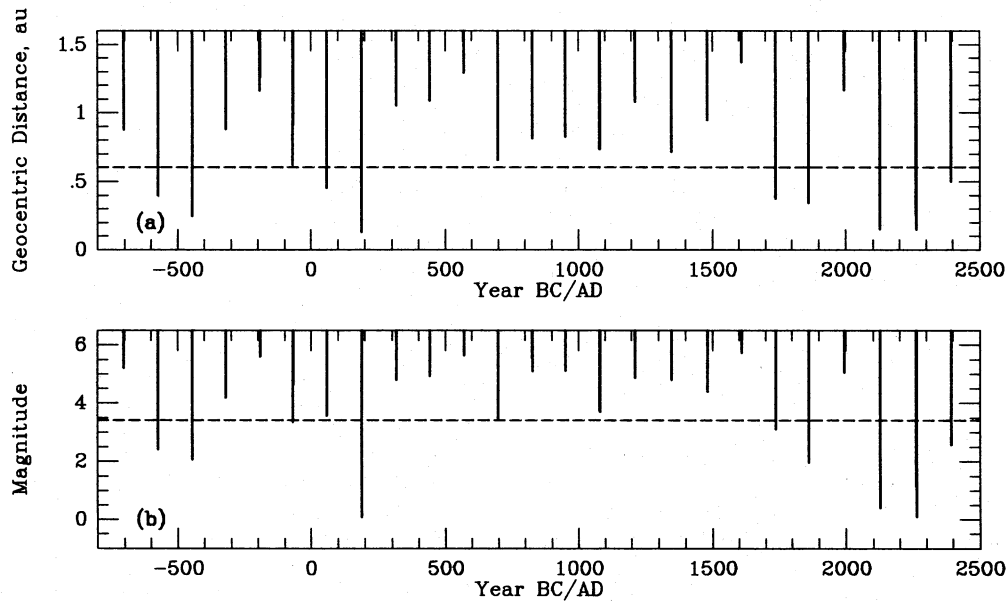
## 5 DISCUSSION AND CONCLUSIONS

The closest approach distances between Earth and P/Swift-Tuttle for all returns since 703 BC are shown in Fig. 6(a). In general, the comet has to approach the Earth to within about 0.6 au for it to be visible to naked-eye observers. The brightest magnitude of Comet Swift-Tuttle near perihelion, as computed from equation (6), is shown in Fig. 6(b). An active comet, such as Halley's comet, becomes visible to the naked eye when the object reaches an apparent magnitude between about 3.5 and 4.0 (Stephenson et al. 1985). A dotted line is drawn at magnitude 3.4 in Fig. 6(b) to show that Comet Swift-Tuttle was recorded only on those returns where its apparent magnitude was brighter than

about 3.4. The slight differences between the recovery magnitudes of Comets Halley and Swift-Tuttle could be due to the relative activity of the two comets. At discovery, Halley's comet was likely to have a developed tail which would have enhanced its visibility. However, none of the records for Swift-Tuttle, apart from the 1862 return, mentions a long tail. Rather, a nebulous bushy appearance was implied. Without a tail, perhaps it was necessary for the comet to become a little brighter before attracting the attention of ancient sky watchers.

The magnitude information plotted in Fig. 6(b) was computed under the assumption that the comet's absolute magnitude (4.5) has remained constant over two millennia. If its absolute magnitude was half a magnitude brighter in the past, it seems likely that there would be Chinese records of its returns in AD 59, 698 and 1079. Likewise, if the comet was intrinsically fainter in the past, it could not have been recorded in 69 BC. Similar to the situation with comet Halley (Broughton 1979; Hughes 1983; Bortle & Morris 1984), the intrinsic brightness of Comet Swift-Tuttle does not seem to have changed significantly for at least two millennia.

We have found no evidence for significant non-gravitational effects in the motion of this comet. However, Marsden (1992a,b) considered them of such importance that he believed that the gravitational solutions completely fail to link the apparitions. In large part, the disagreement concerns the 1862 October observations from the Cape of Good



**Figure 6.** (a) The minimum distances between Earth and P/Swift-Tuttle for all of its returns to the inner Solar system since 703 BC. The dashed line marks a geocentric distance at 0.6 au. (b) The brightest apparent magnitudes of P/Swift-Tuttle around the time of perihelion passage for all returns from 703 BC to AD 2392. These values were computed from equation (6), and determined by noting the brightest magnitude achieved by the comet in a dark sky. The dashed line represents apparent magnitude 3.4. For Comet Swift-Tuttle to be discovered with the naked eye, it has to be brighter than magnitude 3.4.

Hope. Unfortunately, these observations are the only ones available for that time period. With or without solutions for the standard non-gravitational parameters, systematic residual trends are evident if the 1862 October data are retained. If one forces the orbital solutions to include these observations, then one must try to introduce short-term non-gravitational effects that somehow explain the deviation of these observations and only these observations. Our approach has been to consider the 1862 October data as discordant, and to eliminate them completely from our orbital solutions 1 and 2. Marsden (1992c) noted that the post-perihelion observations in 1993 should be examined for the same type of systematic residual trends that are evident in the 1862 October observations from the Cape. We have done so, and find no systematic residual trends in the 1993 post-perihelion observations.

At some level, the motion of Comet Swift-Tuttle is certainly affected by non-gravitational accelerations. We found that orbit solutions over the 1737–1993 interval were more successful if we solved for the radial and transverse non-gravitational parameters. Yet the formal uncertainty on the radial parameter is larger than the value of the parameter itself, and the value of the transverse parameter ( $A_2$ ) is an order of magnitude smaller than for Comet Halley. In addition, if we initialize a long-term backward integration using our orbit 2 (with non-gravitational effects included), it is apparent from Fig. 1 that the comet's predicted motion in AD 188 and 69 BC is not consistent with the Chinese observations.

It is instructive to compare the dynamics and physical behaviour of Comet Swift-Tuttle with those of Comet Halley. Like Comet Halley, the absolute magnitude of

Swift-Tuttle has remained relatively constant for two millennia. This absolute magnitude is about 4.5, a figure that is about one magnitude brighter than the pre-perihelion value for Comet Halley. In addition, the gas production rates for both comets are comparable for the same heliocentric distance (A'Hearn, private communication). Of course, Comet Halley outgasses more after perihelion than it does before, and this not only makes the comet brighter post-perihelion but introduces a non-gravitational effect that increases this comet's period by approximately four days at each return. Since the non-gravitational change in a comet's orbital period varies as the square of the orbital period itself, Comet Swift-Tuttle would suffer an orbital delay of 12 d if it had the same mass as Comet Halley and the same outgassing forces around its orbital path. Yet the non-gravitational parameters of Swift-Tuttle are negligible when compared to Comet Halley. It seems likely that the mass of Comet Swift-Tuttle is larger than that of Comet Halley, so its motion is little affected by non-gravitational forces. From their estimates of the masses of the meteor streams associated with each comet, Hughes & McBride (1989) also concluded that the mass of Swift-Tuttle is considerably larger than that of Comet Halley.

If we assume, for the moment, that the mass of Swift-Tuttle is not far larger than that of Halley, the constancy of its absolute magnitude and the absence of significant non-gravitational effects place strict constraints upon models of the nucleus of Swift-Tuttle over its entire observational period of two millennia. If Swift-Tuttle is not overly massive, then it seems likely that (1) the size and activity of its outgassing areas have been relatively constant, (2) the obliquity of the rotation pole has not evolved over significant

angles, and (3) the outgassing is directed radially toward the Sun, and this outgassing acts symmetrically with respect to perihelion. If these three constraints are not met, the noted stability in the comet's brightness and the absence of significant non-gravitational effects would not be possible. One could envisage scenarios where active vents die out or spring to life in such a fashion as to compensate for changes in the obliquity of the rotational axis, or scenarios where the comet outgasses isotropically so that no non-gravitational accelerations are evident. However, without special circumstances or a very massive comet, these three constraints would have to be valid. The validity of all three constraints over two millennia seems less likely than the much simpler hypothesis that Swift-Tuttle is too massive to be accelerated by outgassing forces.

On account of the long-term stability of the comet's absolute brightness, the negligible effects of non-gravitational accelerations, and the ability to represent accurately the comet's past motion over more than two millennia, we can confidently predict the comet's future motion. We integrated the comet's orbit forward in time in the same manner as described in Section 3, using the DE102 ephemeris. The future returns of the comet and close approach distances to the planets are shown in Tables 3 and 2 respectively. We note that during the comet's next perihelion passage on 2126 July 12 the comet will pass 0.153 au from the Earth on August 5, reaching an apparent magnitude of 0.7. While the comet's appearance will be visually impressive, there is virtually no chance that the comet will collide with the Earth in 2126 August (see also Marsden 1992c). The following return, in 2261, comes even closer to the Earth, reaching a minimum distance of 0.147 au, but again the Earth will remain safe from collision.

#### ACKNOWLEDGMENTS

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